

Forward

The Independent Learning Assignment Presentation Evening is, and remains, the highlight of the year for RGS Scholarship. In short, the ILA is an extended researchbased project which Sixth Formers undertake in the summer between the Lower Sixth and Upper Sixth. Following a rigorous period of marking, each department nominates the two best ILAs from their department which are presented to a committee. Following heavy scrutiny, ten Upper Sixth Formers are offered the opportunity to present their ILAs/CREST project to a live audience. It is one of the few opportunities during the school year that STEM subjects are presented alongside the Arts & Social Sciences.

This year, the spectrum of ILAs and CREST projects on offer was broader and stronger than ever. For example, one boy wrote an essay examining the effects of the Grime movement on UK Politics, whilst another investigated the effects of differently designed javelins on their usefulness on the athletics field.

Although we justifiably should recognise the winners of the ILA Competition, Sam Jones (Arts & Social Sciences) and Ed Ferguson (STEM), everyone who completed any work of scholarship last summer should feel equally proud in the work they produced.

For this year's Journal, we have chosen to publish the ten students whose ILAs were selected for the ILA Presentation Evening during the Michaelmas Term and one exceptional CREST Project. I would warn you, most of us still struggle to understand half of what the ILAs are talking about, that's how complex the ideas and thought processes are! With that in mind, I am proud to introduce the ten students:

From the STEM category: Ed Ferguson (Biology and Physics), Cameron Salter (Physics), Mikel Bober-Irizar (Computer Science), Henry Thake (Maths) and Gabriel Johnson (Biology).

And from the Arts & Social Sciences category: Sam Jones (Music), Thang Tu (Classics), Abhay Gonella (Economics), Kit Edgecliffe-Johnson (Politics) and Harry Dennis (History and English).

To the members of the Lower Sixth, don't worry! If the ILA appears to daunting a task to take on, find something that interests you, do a little research and you should find an idea. It took me three attempts before I had the ILA I actually submitted! My greatest advice is do whatever interests you the most, that is the idea behind the ILA...whether that be the History of Algebra or creating a short documentary. All works of scholarship are welcomed and judged equally.

Once again, thank you to all members of the Upper Sixth for their continued efforts. Thank you to the boys who have allowed us to publish their ILAs. At the end of reading these we hope you are intellectually stimulated, have an appreciation of the works of scholarship undertake by our gifted Sixth Formers and have gained a better understanding of how to make the perfect taffy... (not to be confused with toffee, obviously).

Harry Dennis, Member of the Senior Scholars' Council

Editorial

The Independent learning Assignment is the pinnacle of a student's academic career here at the RGS. It is also the first time they have chosen their own homework. As such their singular passions are on display and it such a pleasure to see the variation of studies undertaken. Some challenge conventional wisdom, some aim to make a difference, some try and solve a problem, and others simply produce works of art or pieces of esoteric academic wizardry undertaken just because the individual derives joy from the struggle. However, and undoubtedly, all are works of great individual scholarship. In reading them I hope that you are astonished that they have been written by 17-year-old young men and then immediately hopeful at what a difference such men can make. I know I certainly am.

Chr. Jonelford

Mr Christopher Bradford Head of Scholarship



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The effects of material properties of javelins on flex in flight

Cameron Slater

ABSTRACT:

Javelin throw is seen as a relatively simple event, while in reality is heavily reliant on technique, skill and power as well as picking out the right javelin. To do this it is essential to understand the workings behind the javelin and how different designs, different weights and different gradings all affect the overall throwing performance. Through a series of tests and closely observed throws, each being critically analysed and recorded, I will discern the best combination of properties after discovering what effect each property has on the overall performance. For the material used I will calculate the stress and strain imposed on the material and in turn the Young's Modulus. From that I will obtain values for flex and compare my calculated result to the given manufacture ratings. Finally I will use slow motion imaging to measure the flex of the javelins when thrown to once again compare measured and claimed values to real measured values. If I have time remaining I will map each throw and represent the projectile motion of the Javelins. I expect that higher gradings of stiffness will help focus the energy transferred to the javelin by the thrower and help maximise the efficiency of energy transfers. Javelins that flex more act as natural dampers and thermal energy transmitters. These observations will then lend themselves to choosing the correct javelin for newer throwers or more experienced throwers and I will also explain why the aforementioned differences are required.

1. INTRODUCTION:

The javelin has been around for centuries, featuring in even the very first Olympic Games back in Ancient Greece. Since then the javelin has been reworked and redesigned to maximise the effectiveness of it.[1] The most recent rework was in 1986 when the centre of gravity was adjusted by 4cm to force the javelin's nose down faster. This change was required when the men started throwing upwards of 104.80m - the new world record set by Uwe Hohn of East Germany and endangering officials on the far side of the stadium. The many aspects of a javelin that determine its nature each have a small impact on the way it performs. The purpose of this project is to investigate and review types of javelin and how the material properties will affect the flex in flight and in turn the distance it is possible to throw the different

gradings of javelin. Having previously thrown javelin myself, the way that javelins can be catered to individuals caught my attention, especially as this customisation helped greatly to reduce my injuries in my early throwing days. I deemed the study necessary after multiple close friends first attempted javelin and started on too high ratings and injured themselves as a result. I was determined to understand the inner workings in more painstaking



detail so I can prevent things like this amongst my friends in the future.

2. TYPES OF JAVELIN:

Javelins predominantly fall into two main categories [2]: Headwind and Tailwind, the main differences being the difference in diameter. Headwind javelins have a larger diameter, typically 2inches, while Tailwind javelins have a 1.5inch diameter. This larger diameter allows the Headwind javelin to "fight the wind" and is the more favoured model overall because the turbulence and wind will have less of an effect than on the Tailwind javelin. With this in mind, I will focus my study around the more popular Headwind javelin. Within each of these javelin types are variations in weight and shape and length. Male javelins typically measure 800 grams and 260cm while the female counterpart measures 600 grams and 220cm. A continuity across all javelins is the shape of the shaft, with a hollow shaft to maximise surface area and minimise weight to increase overall flight duration. Most competitive javelins are

constructed from Aluminium and steel with the occasional carbon fibre shafts as theses are much more durable in order to survive the massive forces and conditions. Common training javelins are made from birch wood, as this is a much more supple material and can flex much more easily. On top of these properties, the binding on the javelin can play a key role. These bindings allow athletes a better grip and more efficient transfer of energy. Usually these bindings consist of tight leather threads and wrappings but some also have a cloth and rubber binding underneath. As these bindings are tightly wound, the javelins tend to snap in this area in the unlikely event one does fail.

2.1 Alterations:

Over time as techniques in javelin have progressed, the distance people are able to throw have increased alarmingly and have resulted in numerous alterations to both the structure and balancing. When it was originally introduced into the sporting world in Greece in 500 BC, The design of the javelin differed greatly from what most people have now come to expect, with the athlete holding onto a thin leather cord wrapped around the middle of the javelin, rather than the javelin itself. When the javelin was thrown the leather cord would unwind and the javelin would follow a spiralled flight with the extra leverage afforded by the cord projecting the javelin further. The javelins, made from thin pieces of wood, were very unbalanced. The rotation stabilised the javelin in flight. In the 1906 Olympics, having been reintroduced, there were very few rules and so a wide range of self-constructed javelins were used. However, firm regulations were set in place by the 1912 Olympics. As I have already mentioned, after an official was nearly injured by a 104.8m throw, the centre of gravity was adjusted forwards by 4cm. The tip of the javelin was also bluntened to make it less aerodynamic. These changes brought the nose down sooner and greatly reduced the flight time. [3] This graph shows how the alterations had effects on the record and medal positions in the Olympics. Designers began to tamper with the official design, drilling dimples and grooves to channel the air and increase the air resistance and drag on the tail of the javelin so the point stayed higher for longer.

2.2 Headwind:

Headwind javelins feature a streamlined tip designed to help cut through the wind. This javelin tip, because of its small surface area, will have self-correcting characteristics when a thrower misses the ideal angle and vector, adding valuable distance to a missed throw. The headwind version is ideal for 'power' throwers who lack perfect technique but make up for it with strength and torque. Oddly, Headwind javelins are still favoured by most throwers as they are ideal for all conditions because the narrower tip is able to "fight the wind" more effectively. This however was a huge myth in the throwing community as the designer of the new 1986 javelin Dick Held [4] confirmed in a questionnaire recently that contrary to popular belief, the Tailwind javelin is actually more viable when the wind is coming from any direction. This is because that even when throwing into Headwinds, there is no guarantee that the Headwind javelin will always land point first unless thrown at a low angle, resulting in a foul throw. This is not ideal and explains why Dick is an advocate for using Tailwind javelins at highly competitive levels, as these athletes present are consistently throwing at the correct high angles. Whereas junior athletes need not concern themselves as they can consistently hit 80m throws while at low angles, allowing for the upthrust and air currents to give the javelin a floating flight.

2.3 Tailwind:

A tailwind javelin possesses a thicker, more blunted tip that increases the surface area in front of the javelin. When a thrower hits the correct vector with a tailwind javelin, it will mimic the old rules style javelins, with a beautiful floating flight and greater distances. This tailwind version is ideal for 'finesse' throwers who lack high outputs of power, yet throw at a high technical level. It has become apparent that more competitive throwers prefer the tailwind javelin as when it is thrown with a slower speed and at a higher angle of attack, the reduced speed in relation to the air results in a lower pitching moment. This reduces and delays the nose diving effect which prolongs the flight, a key feature with the introduction of new rules javelins. Another favoured technique was to throw at a relatively low angle (<30°) and at extremely high speeds (>30ms-1) which would result in a large updraft that carried the javelin in a low arcing flight.

3. YOUNG'S MODULUS:

One of the core material properties of any product is the Young's Modulus which combines the tensile stress and tensile strain to determine the brittleness and ductility of a material. I got in contact with a javelin design brand called Nordic - a notable leader in the field of research - and asked them if they would send the results from their testing of batches of javelins. From the data they supplied, I was able to calculate a Young's Modulus. I predicted that the javelins would have a relatively stable gradient as the javelins are both stiff and tough. I have calculated values and have found my result to be surprisingly close to the accepted value [5] of 69-72GPa. My average calculated gradient was 68.3GPa which I then immediately checked against the following equations.

Thankfully my predictions of a tough and stiff Young's Modulus held true as seen by the gradient in my graph below, these properties are highly desired in all javelins.

$$E = \frac{FL}{\Delta LA}$$
$$F = \frac{EA}{L}\Delta L$$



The essential part of this being that the javelin does not deform plastically and does not creep, instead snapping outright, as demonstrated by my test javelin. This elastic extension is essential for the longevity of a javelin as the continual impacts slowly decrease the javelins structural integrity until at a fracturing point. I will later use the value of Young's Modulus for my calculation of my maximum deflection with Moments of Inertia. One Slight error that arose was that my javelin is not a constant tube and so I had to factor in the area of the cone narrowing as I progressed down the javelin and so I had to include a factor of one over root three in my calculation.



3.1 Stress:

There are two key variations of stress, Engineering Stress and True Stress, the key differences being the modeling assumptions made. Engineering stress makes assumptions about the area a specific force is acting upon, assuming the area remains constant. However, when a force is applied, the test area will inevitably stretch and increase. True stress is aware of this and takes such factors into account. The most evident occurance of this is "necking", where the beam or tubing under test begins to narrow and kink in a certain location. This location is known as the neck. The True stress will be considerably larger than Engineering stress as the Engineering stress is calculated by dividing the applied load by the initial cross sectional area, while True stress is the applied load divided by the resultant cross sectional area.

3.2 Strain:

While stress is the load per unit area acting within a material, Strain is the change in shape of an object in response to external pressure or internal stress. To complicate matters, strain causes the transmission of stress through an object. As in, the strain causes an internal "movement" causing one part of the inside of an object to press against the material next to it generating stress in this region, this in turn can cause more strain and so on. There are a number of differing types of strain, for example axial strain is the change in length relative to the original length of an object, this change in shape is also known as deformation. However, Volumetric strain occurs when an object is squashed or pulled on all sides leading to a change in volume.

4. FLEX:

Most importantly regarding my investigation is Flex, before understanding flex, it is essential to first understand the other factors in material properties as described above. When a javelin is thrown at an incorrect or inefficient anale, in order to reduce the stress and lashback on the thrower's arm and elbow. the javelin flexes to absorb and later release the extra energy as thermal and kinetic. Each material of javelin is made slightly differently with the aim of varying the flexibility. The variation in flex is largely due to the material properties of a javelin and the way in which they are constructed. I elected to test the flexibility of a recently snapped headwind javelin with a weighting of 600g and made of Aluminium. Flex is nominally used to give a rough estimate as to how far a javelin could be thrown but again, this all varies with technique and power. Some companies encourage and incorporate flutter and spin into their designs as the less stiff javelins are actually preferred by some athletes. While it is not entirely clear what exactly causes the flutter, it has been narrowed down to either the axial spin or the transfer of longitudinal energy in the pull down motion when launching. It is estimated [6] that javelins can be accelerated up to 40Gs and that the large accelerational forces cause the flutter. It has also been calculated that most competitive 800g javelins flutter with a first harmonic frequency of 24Hz, calculated using the equation below. [7] I will compare that value to a measured value of my own, using my javelins full length of 2.21m, density of 2712kgm-3, Bulk Modulus of 70GMPa and Shear Modulus of 27MPa. I get a resulting frequency of 43.6Hz, which is almost double the frequency of the 800g javelin.

$$f = \frac{1}{2l} \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

where K = Bulk Modulusand $\mu = Shear Modulus$

However a stiffer javelin will vibrate with a smaller amplitude as the energy is more focused and less is wasted in axial rotation. Flex is spoken of in broad sweeping statements but typically in the world of throwing, the athletes and coaches are referring to the softness and stiffness. There are a variety of scales to measure this flexibility but most are comparative and largely ineffective such as Nordic's new addition on a scale of 0-20 with 0 being the stiffest and 20 being the softest. Now one might assume that due to the vagueness of the measurability, flex is a pretty irrelevant part of quantifying javelins. It is however of

the utmost importance in my opinion, as the more technically demanding stiffer javelins can be extremely punishing to first time throwers. Fortunately there exists the Meter Scale which gives a rough idea of what type of javelin you should be throwing depending on how far you are currently throwing. One of my main aims of my investigation was to measure and then test the validity of some values for maximum deflection in my javelin. I started by clamping my javelin to a workbench and marking Displacement_o. I then proceeded to slowly load the end of my javelin and measured the deflection for each load. In order to calculate δ max I first needed a value for Young's Modulus, calculated and checked against accepted values further up the document, and Moments of Inertia using the equation below.

$$I = \frac{\pi (d_o \ ^4 - d_i \ ^4)}{64}$$

Where $d_o = cylinder \ outside \ diameter$ $d_i = cylinder \ inside \ diameter$

Which can then be combined into [8] the equation for Deflection_{\max} when I is the Moment of Inertia in the javelin.

$$\delta_{max} = \frac{Pl^3}{3E(\frac{I}{\sqrt{3}})}$$

I had to include a root 3 in my calculation as mentioned before as the diameter of my javelin tapers down to a narrow point and so I cannot model my javelin cantilever as a uniform tube. Instead I am likening my javelin to a combination of a cylinder and cone. Comparing the equations and simplifying both volume equations pares down to that differing factor. I once again found that my measured values lined up perfectly within 1mm error margins for my Force against Deflection_{max} when plugged into my equation which confirms my values for Young's Modulus and Moment of Inertia.



4.1 Meter Rating:

This rating [9] references how stiff and forgiving the javelin is. The ratings usually range from 35 meters - 100 meters. The meter rating indicates the optimal

distance for a javelin to be thrown to land point first at a 5° angle. The scale takes your current personal best and from there, depending on what javelin you threw that personal best with, suggests other types of javelin from Aeroflo to OTE, two different brands, to the correct flex rating. This range is varied by the weight distribution in the javelin which needs to hold the forces applied through the javelin. This means the javelins with higher meter ratings need to be stiffer for optimal energy transfer. The lower meter rating javelins are also favoured for newer throwers with questionable throwing styles as the flex is much greater and allows the javelin to fly better and decreases the amount of stress through the elbow. In higher tiers, the stiffer javelin applies more force through the elbow and a more accurate and smoother throwing style is required to negate injury. This lack of flex means the javelin will fly in the exact direction the tip is pointing in, and as a result will lose distance to rotational errors.

4.2 Technique:

In the run up, the main aim is to build momentum. This momentum is then almost entirely conserved and passed into the javelin. The rapid acceleration being much larger than the airspeed results in a high soaring flight. An important technique point is to rip the hand down and whip the fingers forward. The ripping action results in the rapid axial spin while the wrist flick causes the longitudinal energy. To counter the flexing of the shaft in flight, the thrower can apply spin to the shaft by rotating the shaft on release, this however is only an acceptable practice and a good idea if the thrower in question can consistently throw "through the point" - a fancy term for "the correct angle of attack.. The rotation of the shaft counters any perpendicular vibration and it makes the javelin more stable in the air. An competitive level javelin thrower can cause the javelin to spin at a rate approaching 25 rotations per second.

4.3 Materials:

Javelins are typically made from either Aluminium, Steel, Carbon Fibre or Aluminium Alloys. Of the numerous javelin manufacturers out there, each has a preferred material of javelin depending on their target market. [10] Aluminium javelins tend to be a lot more flexible and the preferred choice for the majority of beginner javelinists with a dodgy technique. Steel is then the next step up, being slightly stiffer and heavier than aluminium, these are preferable for junior or senior athletes coming into or just out of their prime. This is because the steel typically weighs in at 600g-700g and so is a very high tier javelin. Steel also comes a lot thicker and heavier than Aluminium so has to be used in thinner dimensions. Despite this, it will flex less in flight due to a larger mass per unit length which results in a smaller fundamental frequency. One unfortunate drawback is that steel is much more susceptible to rusting and so are less price efficient, having to be serviced or



replaced much more frequently. Carbon fibre shafts are a lot more rigid and efficient at transferring energy as the woven fibres dissipate energy all over the javelin. Because of the production process and intensive labour, the final price are much steeper than full metal javelins. The grips of javelins are also varied in material but mostly depend on the material of the actual shaft or just personal preference. Cotton grips are most popular due to their effectiveness in all weather and climate conditions. Synthetics, however, are less effective in wet conditions. They are ideal for more flexible javelins though, as they have lots of give in them for the inner flexing of the javelins.

4.4 Weighting:

One of the most important design and manufacturing processes is balancing the javelin, the centre of mass now has to officially be located 40mm in front of the centre of pressure. This will affect the rotational forces acting on the javelin and cause the javelin to rotate and fall back down to the ground much sooner. The centre of pressure is the combination of the aerodynamic forces of drag and lift acting around the javelin. Most javelins use solid tips as a way to create this balance, but the most common methods in older javelins was to combine lead weights, glue and string. However, these combinations tend to come loose and rattle which renders the javelin illegal.



The Bernoulli effect is taking effect in javelin flight, depending on the angle of attack. The higher the angle, the larger the pressure below the javelin, resulting in a long flight time. All this is dependant on the initial speed of the javelin throw, creating the fast moving airflow over the top of the javelin and the slower flow underneath which generates lift. As the javelin approaches the peak of the arc, the velocity is momentarily zero. And so, the lift stops acting and the nose rotates down so the air now moves slower over the top of the javelin so the javelin is forced downwards.

5. PROJECTILE MAPPING:

I have used a programme which was originally intended for developing an understanding of the correct type of javelin to throw to model the motion of a javelin through the air having been thrown with a constant force and speed, relaying the resulting distance. [11] I used a constant force of N and an initial speed of 28ms-1 which is standard speed for most elite throwers and being released at an angle of 32° which is considered to be the optimum angle of attack.

5.1 Figure 1:

With conditions remaining constant, I proceeded to simulate the flight paths of both javelins and record both the flight arc and deflection. I used standard aerodynamics of a tailwind javelin in terms of the centre of lift and centre of drag. The power I set as a constant of 70N which is also typically how much power is supplied to a javelin in a throw. I made sure to throw each of these javelins with a 30° delivery angle but a 10° angle of attack resulting in a $\pm 5^{\circ}$ angle either way of the release angle to allow the effect of flex to take place if at all.



With an 800g men's competitive javelin, I simulated that the stiffer 100m rated javelin would fly around 92.5m in a reasonably straight line and constant curve.



My simulation of a lower rated, 600g 50m javelin, such as the one I tested, found that the javelin would fall 20m shorter and veer off to the right as a result of the flex.

6. CONCLUSION

The large variety in javelins in terms of mass and stiffness and design can lead to an overwhelming confusion when a new athlete starts to pick up the sport and was certainly a dissuading factor when I first started throwing. On top of this, throwing a certain javelin with poor starting technique can cause permanent injury. The aim of my research was to discover what causes the stiffness, how it affects the flight and how it is tailored to the thrower in particular in an effort to reduce the frequency of injury. I have discovered that the different flex gradings of javelins on the meter scale are are varied by the type of material used, more specifically, the density, shear, Young's and bulk modulus. I checked this by comparing the known value to my measured values of maximum deflection and my value obtained from Nordic Javelin's quality control testing.

On top of this, I have analysed the effect of The Bernoulli Effect on the flight of the javelin and how the forces of lift and drag have an impact on the javelin's flexing. I have also looked at how the design of a javelin varies the throwing technique and therefore the flight. My results followed the accepted values for each material property and I was able to deduce from my modelling that the tailwind javelin with a greater flex meter rating will fly further than a more flexible javelin which has been proven by olympic athletes time and time again. In addition, I was able to work out the energy dissipated by a lower grading of javelin was much larger than the energy lost by a javelin with a higher rating. There are limitations of the study, such that I did not have a wide range to test and insufficient means to test them. On top of this, I had to model my javelin as a regular cylinder which had a slight impact on my readings, in order to counter this I used a constant to scale my javelin into the likeness of a cone. From all my research and testing, I have discovered that the stiffer and more heavier javelins will be less susceptible to flex when thrown by my simulated data. It is clear to see how the more flexible javelin flew a shorter distance and yawed off the straight path, while the stiffer javelin had a longer flight and more direct pathway.

Measuring the dynamic strength of metals using the reverse Taylor impact test

Charles Dorkins supervised by Dr Daniel Eakins of the University of Oxford

Introduction

The Taylor Impact Test is a material test which involves firing a cylindrical sample of a material into an anvil at high speeds. This projectile impacts the flat anvil perpendicularly sending first an elastic wave then a slower moving plastic wave through the sample. The elastic wave reaches the end of the sample, where it is reflected back towards the impact site. At a certain point, which depends on the speeds at which the waves travel through the sample, the two waves will meet. The area behind this boundary will not have experienced plastic deformation as only the elastic wave has passed through it. The Taylor impact test is somewhat similar to a car crash test in which a car collides with a wall head on. Despite some differences a car crash test is a good way to visualise what is going on.

This experiment was designed by Sir Geoffrey Taylor in 1948 and is very effective as the sample that is impacted undergoes dynamic loading as the impact end experiences substantial plastic deformation; but the end furthest away from the impact site ordinarily does not undergo any plastic deformation. The rest of the sample undergoes varying amounts of plastic deformation. The main drawback of the Taylor Impact Test is that although high speed imaging allows for photos to be taken during the impact and measurements can be taken on the sample post impact, very little data can be recorded about the sample during the impact. This was not a problem when Taylor first devised the test as the technology was not good enough to measure anything accurately over such short periods of time.

More data can be recorded if the setup is altered so that the anvil impacts a stationary sample rod. If we reconsider the car crash test, this would be like leaving the car stationary and moving the wall towards the car. This is known as a Reverse Taylor Test. In this way the sample can be better monitored, using a PDV (Photonic Doppler Velocimetry) probe or VISAR (Velocity Interferometry System for Any Reflector) system. The main disadvantage of this Reverse Taylor Impact Test is supporting the sample. If the supports absorb some of the shock, then the resulting (deformed) sample will be altered. The picture on the cover shows high speed images of a Reverse Taylor Test. The individual frames show the projectile approaching and impacting the sample. The plastic deformation is just visible to the naked eye.

The samples in both the Taylor and Reverse Taylor Tests undergo plastic deformation at the impact site. The exact shape depends on the material and the impact velocity. There are mathematical models to predict the shape and size of the plastic deformation at the tip of the sample (such as the Johnson-Cook or Zerilli-Armstrona models). Generally, as the total volume of sample must remain the same, the faster the impact the greater the difference between the original length and final length, and the greater proportion of the final sample undergoes plastic deformation. Together, these mean that the length of the deformed area does not increase in a linear manner like one might imagine, but in fact it increases to a point and then begins to decrease when the velocity is increased. This is because although a greater proportion of the total length undergoes plastic deformation, this total length also decreases, meaning the actual length of the deformed area starts getting smaller after a certain impact speed.

PDV or Photonic Doppler Velocimetry is a technique used to calculate the velocity of a projectile, typically travelling faster than 1000 m/s. The technique involves sending light onto the target, this light is then doppler shifted (as long as the target is moving), this doppler shifted light is then collected and sent to a detector, at the same time an equal amount of none doppler shifted light is also sent to the detector and a beat frequency is calculated. It is then possible to calculate the velocity at which the sample is moving from the beat frequency.

Fig 1 is a diagram showing how the PDV probe works. In the diagram f_0 is the initial frequency and f_d is the doppler shifted frequency (here $f_d > f_0$ as the surface is moving towards the probe), the beat frequency or f_b is equal to f_d - f_0 . The greater the velocity of the surface, the greater f_d and consequently the greater f_b .



We can use f_b to calculate the velocity of the surface by simply using the equation $f_d = f_0(\frac{2v}{c} + 1)$ which rearranges to give us $v = \frac{1}{2}f_b\lambda_0$ where λ_0 is the original wavelength.



Above is a simplified diagram of the setup of the single stage gas gun. It is important to note that all black lines are high/ low pressure gas lines and the breech is a cylinder that wraps around the barrel.

The gas gun uses high pressure gas (approximately 200 atm) to fire the projectile down its 3m long barrel into the target chamber. The gun uses a high-pressure system connected to several valves, 2 vents and 2 vacuum pumps.

After the target is positioned and light gates etc. are switched on and working, the target chamber lid is put on and firmly screwed into place. Next the projectile is loaded into the barrel and it is sealed. Then the smaller of the 2 vacuum pumps removes all the air from the area behind the projectile, at roughly the same time the stronger vacuum pump removes all the air from the barrel, target chamber and overflow tank.

Then the fill valve is opened, and the breech is pressurised. The smaller vacuum pump is turned off and the pressure behind the 'fire' valve begins to build up. When ready to fire, the 'fire' valve is opened, this pressure causes the projectile to be pushed forward. When the projectile is pushed forward far enough, all the pressure from the breech is released into the area behind the projectile (through the 2 slits) causing acceleration in the projectile, this sends it down the barrel and into the target at a very high speed. The highpressure air from the breech fills the barrel, target chamber then the overflow tank. In this way, the pressure of the target chamber never gets above one atmosphere. Finally, air is slowly sucked into the system and then the sample can be removed and examined.

Basic Alignment

In order to ensure that the impact between the anvil and the sample was perfectly perpendicular, lasers were used, and a mirror put on the end of a test sample. The laser was reflected off two mirrors before travelling down the barrel. By using two mirrors the laser could be adjusted so that it travelled straight down the barrel (without ever reflecting off the inside of the barrel). Once the laser had been aligned, the test sample was added. The test sample is usually held in place by two supports, which were embedded in a 5-axis mount, however for the purpose of learning how to align the 5-axis, I used a mirror instead of a sample. The 5-axis allowed for fine adjustment over the sample to ensure it was properly aligned.



These images show the double mirror setup which sends the laser (shown by the green line) down the barrel. At the other it reflects off the mirror mounted the 5-axis and travels back up the barrel. Back at the original (breech) end of the barrel an iris or simply the mirrors are used to ensure the mirror is aligned perfectly.

NB: The picture on the left shows the laser travelling through a beamsplitter then a mirror, but for the purpose of this alignment, two mirrors would be sufficient.

Misalignment/ Error

After the mirror and laser were aligned, the next task was to design a method to try and quantify any error in alignment caused by; firstly, reducing the pressure of the barrel and the target chamber to a vacuum and secondly (for a more advanced experiment) heating the sample using two halogen lamps. Any misalignment caused by vacuuming down the barrel and target chamber was expected to be minimal; however, it was possible that in heating the sample to high temperatures, the sample might become misaligned slightly and so it was necessary to design an experiment to try to find an approximate value of this misalignment. To do this, the setup was altered slightly. The first mirror was replaced with a beam splitter and another mirror was added.



This now meant that the laser (shown as a green line) went through a beam splitter and then reflected off a mirror; it was then sent all the way down the barrel to the target chamber where it was reflected off a mirror (although in the case of heating the sample, a test sample with a mirror attached using special heat resistant glue would have been used). This mirror would be perfectly aligned (pictured above), so that the laser would then be reflected back through the barrel, travelling in the opposite direction. At the end of the barrel it would once again reflect off the mirror, but this time some of the light would pass through the beam splitter and reach another mirror (In the above image this is represented by the red line). Here it would be reflected parallel to the barrel all the way down to the target chamber casing, where the position of the beam was marked onto a sheet of paper on the side of the target chamber. The laser now travelled a length of over nine metres in total and the distance from the mirror to the marked reference point was over six meters. The angle θ (angle of misalignment at the mirror/sample caused by heating/vacuum) would be measured using simple trigonometry and for this reason the longer the distance between the mirror/sample and the marked reference point, the greater accuracy θ could be measured.

Upon heating/vacuuming, any shift in the centre of the laser beam would be recorded. The distance between the centres would then be measured to allow θ to be calculated. Although in the picture the laser beam is not perfectly focused on one point, a better focus could have been achieved by using higher quality plastic at the end of the barrel; however even with this size of laser dot a movement of 0.5 cm could be measured. This would correspond to an angle θ of approximately 0.8*10⁻³ radians. This is an incredibly small 2.7 arc minutes (45*10⁻³ degrees).



HEATING APPARATUS

The aforementioned heating apparatus was to be used to heat up the samples during the shot, allowing data to be collected on how the material impacted at higher temperatures, this would be very useful in situations in which the material would be operating at temperatures above room temperature, for example in an engine. It used two halogen lamps which were positioned around the sample, then the lamps and sample were covered with a layer of metal shielding to minimise heat loss to the surroundings. Both lamps required a mains, earth and neutral wire, these were passed through a small window in the target chamber, then sealed with vacuum putty. As well as the lamps being grounded to the target chamber, the metallic shielding and the frames supporting the lamps were also grounded to ensure the system was safe.





The heating apparatus with and without the metallic shielding, both of these have a 5-axis mount underneath the frame. In the photograph on the bottom (with the shielding) it is noticeable that there is a small window between the two pieces of shielding. This allows for any high-speed imaging of the sample.

Analysis

LIGHT GATE SIGNALS

At the end of the barrel; inside the target chamber and only a very short distance away from the target were two beams of light travelling across the barrel. These beams entered the barrel on one side from a fibre optic connected to a laser and left on the other side of the barrel in another fibre optic which was connected to a detector which recorded the signal strength of the laser. These beams of light were separated by a known distance (in the order of a few centimetres). When the projectile reached the first beam of light, the signal strength of the first laser records a large drop as the projectile prevents the signal from being transmitted by physically breaking the beam of light. The projectile then continues on and breaks the second light beam, causing the signal received to fall to a near zero value.

This is a plot of the light gate signal strength (before



being normalised). The blue line is the first light gate and the red line is the second light gate. The two plots are very similar; however, the second light gate simply has a small offset. The only discrepancy is the spike in both, which was due to a flash that was independent of the projectile, causing both detectors to measure an increase in signal strength simultaneously.

By knowing the distance between the two light gates and the time between each one being broken it is possible to calculate the velocity of the projectile using speed = displacement/time. The only problem with this is that the light gate signal has a lot of 'noise' meaning it is unclear as to when the beam is actually broken and when the signal first starts to drop off. To get around this, the average of the signal was found which reduced the noise significantly, then the data was normalised meaning that the y axis now went from ~0 to ~1 instead of the actual signal strength. Furthermore, instead of using the point at which the signal strength initially drops, it was more accurate to use the points at which the signal strength is halfway between the average of the strength before the beam is broken and the average of the strength after the beam is broken as these two points are much clearer and easier to find, this means that the time values found have less error in them meaning the final velocity calculation is more accurate.

To find the time value when the signal strength was at its halfway point, a complementary error function was fitted to both sets of the light gate data. In this complementary error function (y=a*erfc(b*(x-c)) + d) the values of a, b, c and d were all found by using the getrect tool in MATLAB which involved the user selecting the area of the

light gate signal and it delivering information about the area which allowed a, b, c and d to be calculated; however this was not perfectly accurate as the area was selected by the user and consequently the values of a, b, c and d were not deemed accurate enough, and the complimentary error function did not fit the data well enough.

For this reason, in order to calculate the impact speed a much more complicated function, which was not written by me, was used. This function started out similar to the one written by me, however it used a more complicated and more advanced method of calculating a, b, c and d and therefore plotted a much more accurate complimentary error function for each of the light gate signals and provided an accurate value for the velocity of the projectile. This function provided a velocity of around 7 km/s



This graph shows just the area of a light gate signal in which the signal strength drops to near zero in black and the blue line is the complimentary error function that was fitted to the curve. It is clear to see that although there is a correlation, the blue line does not perfectly map the black line and for this reason, this simple approach could not be used to find the time difference.

PDV TRACE

Having found the impact velocity of the projectile, the PDV probe allowed the velocity of the sample during the impact to be measured. Behind the target was a mirror, this allowed the PDV probe to not be directly behind the target meaning it would not get damaged in the shot, but for the light from the probe to still detect the movement of the sample. The PDV trace that was recorded during the shot consisted of the actual signal (the beat frequencies) as well as the time. However, the raw PDV data was in the form of two column vectors, both consisting of one column and 8,000,002 rows, which would be quite awkward to analyse and for this reason the first task was to crop and then reformat the data into more manageable chunks.



This graph shows the PDV signal after it had been cropped and segmented. Each segment in this graph was plotted in a different colour, hence the multi-coloured effect, however it is impossible to make out each individual segment as they are far too small. This PDV trace cannot tell us much about the movement of the sample as it is very hard to interpret in this form, as there are so many data points, large areas of the graph are/appear to be completely shaded. NB: this graph has been stretched considerably in the y axis (signal strength) and compressed in the x axis (data point no.); this highlights the additional noise in the signal strength and makes it impossible to see subtle trends over the data points, but makes the graph have suitable dimensions.

After the data had been transformed into an easier to analyse format (256*31250), it went through a fast Fourier transform. A Fourier transform breaks up a complex signal into a number of simple sine waves of differing frequencies and amplitudes. A simple everyday way to visualise a Fourier transform would be to make a smoothie out of different fruits and then put a sample of your smoothie through a Fourier machine. This (made-up) Fourier machine would tell you which fruits went into your smoothie and the ratio between them. Although a Fourier transform deals with signals, not smoothies, the principle is very similar. In the same way that you would be able to make a smoothie from a list of the ingredients and the ratios between them, a signal for example a song can be remade from the sine waves obtained by putting it through a Fourier transform. The output of my fast Fourier transform function shows the frequencies of the sine wave(s) which make up that 256 data point segment (for this analysis only the real/ absolute part of the Fourier transform was needed as this contains the frequency of the sine wave, whereas the imaginary/complex part of the Fourier transform

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contains the offset of the sine wave, which was not required for this).



This is the Fourier transform for the third segment (of 31250) of the PDV trace above. This shows how the segment is effectively a sine wave with a frequency of approximately 3.5 GHz (this frequency value was calculated through other calculations). This information would have been almost impossible to obtain through the PDV trace alone and now this segment of 256 data points has effectively been reduced to one frequency value, this makes the data a lot more manageable as all the segments can go through the Fourier transform and very little information about the segment is lost.

The advantage of splitting the data into lots of small segments was that the Fourier transform for each individual segment was much easier to analyse on its own as it represented a small range of values, meaning that the Fourier transforms were often just made up of one frequency (the segment was only made up of one sine wave). The resulting Fourier transform was only a transform for 1/31250 of the data and so it would be ineffective at looking at trends across the entire data set. In order to look at all the Fourier transforms from each of the 31250 segments (consisting of 256 data points each) all the Fourier transforms were plotted next to each other in a 3D surface plot.

These surface plots contain the Fourier transforms for all of the segments; the top image is a plan view. The frequency of the sine wave is the x axis, the y axis is the segment number (1-31250) and the colour corresponds to the height of the peak of the Fourier transform (amplitude of the sine wave). The other image is another view of the same surface plot, displaying all 3 axes, this provides a better visual picture of the height of the Fourier transform (amplitude of the sine wave) and may aid in picturing all the transforms next to each other. These surface plots allow us to view all the data in a much more manageable format as it is much simpler and clearer than the original PDV trace. It is also allows us to see subtle trends in the data.



Another way in which the data was analysed was by using a spectrogram. To start with, the built-in MATLAB spectrogram was used to perform a short time Fourier transformation on the data then plot it as a spectrogram.

The first spectrogram is similar to the surface plot generated by my functions. In this spectrogram the y axis labelled 'Frequency' is the frequency of the sine wave that made up that segment; however instead of simply the segment number, the other (x)axis is now a 'Time' axis. This uses the time data from the original PDV data and maps the segments to a time. Finally, the coloured axis labelled 'Power/Frequency' is the axis which gives us information about the height of the Fourier transform (amplitude of the sine wave). As I was only interested in the peaks of the Fourier transforms (as these contained all the information for the segment), this area was enlarged, and the result is pictured below.



Here we can see that the frequencies are constant to begin with, then they rise and at ~70µs some oscillations in the increasing frequency can be seen. After increasing, the frequencies level out for a short period of time; however, then the height transforms of the Fourier (the coloured Power/Frequency axis) records a significant drop, this can be seen in the original PDV trace. There are then 3 small areas of a decent height before the signal drops again. These features can once again be seen on the original PDV trace. These drops are caused by the fact that the PDV signal was very weak at these times, this means that it is very hard to find just one frequency of sine wave.

After this, I wrote my own function which took the outputs from the spectrogram function and allowed them to be examined in more detail. For example, the area of interest in the spectrogram, in which the frequencies from the Fourier transform rise. This jump in the values corresponds to an increase in the frequency of the beat frequencies recorded by the PDV probe. This area was analysed through plotting only the highest x value from each of the Fourier transforms (only the frequency of the sine wave with the largest amplitude per Fourier transform was plotted); this is the same as the greatest z (amplitude of the sine wave) coordinate for each x (time) value on the spectrogram. This is the very similar to plotting the yellow line from the zoomed in spectrogram above. This produced two graphs, the first was simply the height of the highest points (on the 'Power/Frequency' axis) and the other was a graph of the highest points against their frequency.



This graph is of the highest points against their frequency value and is clearly correlated to the spectrogram above; the frequency is constant for a period, it then rises and then flattens off for a short period (around the 8000 data point). After this the graph drops to zero frequently; however, there are several patches of the same frequency as just after the jump.



The graph below explains these sudden drops to zero. It is the plot of how high the peaks are and resembles the original PDV trace somewhat. This graph shows us that just after the 8000 data point mark, the height of the highest peak is very close to zero. This can also be traced back to the original PDV signal in which the signal strength drops significantly. We then see three small peaks, 2 of them very close together (these can also be seen



in the original PDV trace). We also see that between the first and second peaks the highest peaks are also near zero values and after the third peak it is once again near zero values. As these values are so small (in some cases 0) we can tell that these Fourier transforms had no real peaks as the original signal was so weak; for this reason, when trying to find the maximum of these Fourier transforms the small spike at the very beginning of the Fourier transform (with a frequency of 0) was recorded. This explains why the graph of the frequencies of the maximum values drops to zero at points (causing the vertical blue lines). This graph also nicely displays the elastic precursor and then the elastic wave. The first part (up to around 4000) is the elastic wave; the 'mess' after this is the plastic wave.

Using the above graph of the peaks against their frequency (at the top of this page) we can find the initial frequency and final frequency of the PDV trace; this means we have a beat frequency between two beat frequencies. With more time this would allow the velocity to be calculated

Wider Use

In Taylor's paper, he theorised that when the reflected elastic wave reaches the plastic boundary, the area behind this point is travelling at $U - \frac{2S}{\rho c}$ this is equal to the velocity calculated through the PDV trace. Where U is the impact velocity calculated by the light, S is the yield stress, p is the density and c is the velocity of the elastic waves. So, if p and c are known, having calculated U and the post impact velocity, the yield stress can be calculated. Taylor was of course unable to do his calculations in this way as he did not have a PDV probe at his disposal and was not using the reverse method. Taylor went on to make theoretical models of the total post impact length/initial length as well as plastically deformed length/initial length. As well as the formula above, Taylor also derived a formula to estimate the yield point of the sample from measurements taken (the yield point or yield stress is the stress at which a material starts to deform plastically). He used his formula to find the dynamic yield stress of paraffin wax. Paraffin wax had the advantage that after undergoing plastic deformation, tiny cracks in the wax cause it to change from a transparent colour to an opaque white colour; this allowed for easier and more accurate measurements to be taken. As mentioned previously, knowing the yield stress allows the comparison of differently manufactured or differently treated materials as well as different the comparison between material models and the actual material.

Is universal basic income the best way to combat rising inequality?

Abhay Gonella, Corpus Christi Dixit Essay – Economics 2018

To answer this question, we must first ascertain different types of inequality to establish whether a Universal Basic Income (UBI) is the best way to combat rising inequality. I will conduct this debate referring to two types of inequality. Income inequality is a significant measure to evaluate how different demographics, classes, regions and/or countries are progressing in terms of the pay they earn as an average. Gender inequality is another contentious issue which limits a large proportion of the population and in extension the whole economy. Inequality is portrayed in society in different ways. The rise of new automation replaces labour jobs and affects low skilled employment adversely; the social welfare system doesn't provide adequate financial aid to the unemployed, who consequently live under the poverty line. In a world where the richest 8% own 50% of the world's income it's time to change to a system that benefits everyone rather than just the social elite. I believe that the current welfare system isn't sufficient to combat inequality; it's only a remedy to slow the disease (inequality) from rising. A UBI, unlike our social welfare system, would cure the disease by providing a set income to everyone without any conditions.

Inequality is commonly associated with poverty. Poverty is stigmatised by the wealthy; they believe that the poor are lazy, that they choose not to work and that an unconditional income would result in the poor spending more on demerit goods such as alcohol, tobacco etc. Essentially people are influenced by the dogma that the poor make unfavourable decisions. This stigmatism towards the poor and their behaviour is epitomised by a quotation from the former Prime Minster of the UK, Margaret Thatcher; 'poverty is a personality defect'. Nevertheless, experimental evidence proves this statement wrong. A study in two districts of Tamil Nadu, India, showed that the IQ of farmers increased by 14 points before and after the harvest. It was calculated that the harvest equated to 60% of the farmers' annual income. These farmers felt more stress before the harvest, because they were worried about their financial stability more than they did after they received their income. This experiment highlighted that low-income earners are not foolish. Instead it's their circumstances which cause them to make poor decisions under duress - something everyone, no matter their income, is prone to do. Money empowers the poor since financial stability is the difference between someone living below and above the poverty line. Once again, an experiment was conducted, this time in London where 13 homeless people were given £3,000 per annum without any conditions. The results were staggering; 1.5 years later 9 of the 13 had homes and 2 owned their own apartment. Significantly, most people were thrifty, spending £800 on average. This discredits the sentiment of the wealthy that the poor spend their money irrationally. Annually, it costs the social welfare system £400,000 overall to give benefits to 13 people. The UBI's total cost to help all 13 people was £50,000. A UBI scheme would in fact help all members of society as it would reduce the burden of tax payers by cutting unnecessary social welfare costs such as unemployment benefits. Instead, the government's tax revenue could be given as a form of income which would help reduce the number of people living under the poverty line and improve human capital, hence reducing income inequality.

UBI sceptics often pose the argument that the scheme would not be affordable. However, there are strong counter arguments to this position. For example, the UK currently doesn't collect tax on the first £11,000 of income. That's about £100bn being foregone. Land value tax on 5% of land in the US could create an extra \$1trillion. A 0.5% levy on ownership of shares in the top 100 companies would equal to £8bn. There are many options to fund a potential UBI scheme which would in turn help people living below the poverty line by giving them financial stability. Financial stability is key to combating rising inequality. As shown in a different study in Madhya Pradesh, India, a basic income helped women challenge social norms and reduce gender inequality. For example: women were more confident and were willing to commute around villages without wearing a traditional veil to cover their face. The scheme allowed women



to challenge the status quo/social norms by giving them more independence – they were less reliant on male income providers. Furthermore, there was a higher attendance for girls at schools; there were more employed women; there were fewer teenage pregnancies and the average weight of all females increased suggesting that they were able to eat more. The results add up to highlight that a UBI scheme could be the difference in combating gender inequality, especially in developing countries, because it empowers women and other minority gender groups.

The inequality problem that the poor face is related to the fact they have little access to liquid assets; hence in developing countries moneylenders are able to charge high interest rates. We must acknowledge that the poor consume less, because they have less income. Arguably however, unlike the middle classes, the poor borrow less due to their financial instability, therefore they consume even less relatively. But this can be overcome with a constant, unconditional income which would allow them to spend money without the obstacle of high interest rates. A UBI scheme would be the best way to combat rising income inequality, because it endorses a redistribution of wealth from the rich to the poor. Consequently, it diminishes gender inequality as it gives women and other minorities a chance to stabilise their finances. Crucially it would help the rich, albeit with a certain time lag, become richer since the so called 'poor' would consume more thus helping businesses owned by the rich. I believe that it's imperative that the poor receive cash to combat inequality; 'Poverty isn't a lack of character. Poverty is a lack of cash.'.

The economic consequences of a UBI seems to suggest that it would be an effective scheme to combat rising inequality. With the rise of new technology, a UBI would protect low skilled labourers against large scale mass unemployment due to the increase of new technology on assembly lines and other labour-intensive sectors. Consequently, when technology does replace workers they will be more resilient to the change and with a stable income inequality will not worsen. Furthermore, a UBI scheme would promote higher consumption from lower income brackets, thus maintaining a rising aggregate demand. A UBI scheme would allow unemployed people to do unpaid social work which ultimately benefits society and helps reduce inequality. For example, someone unemployed could look after their parents; social welfare costs would decrease as a result, ultimately benefiting the poor who would pay less taxes and have more effective disposable income. These consequences show the positive benefits of a UBI scheme; it combats rising inequality more effectively than current or previous schemes.

In theory trickle-down economics should combat inequality. High government taxation disincentives firms from investing. By lowering taxes - firms should increase their labour force - unemployment decreases. If unemployment decreases, more people live above the poverty line; they consume more which causes a positive multiplier effect where increasing consumption leads to increasing investment and it stimulates GDP growth. The benefits the rich experience should result in a movement of wealth from the top to the bottom. However, in practise the movement of wealth doesn't occur. The rich save as much as they can; they absorb all the benefits. Rent seeking, where wealth at the top of society isn't created but instead transferred to them from the poor, shows that this theory isn't acceptable for combating inequality.

Furthermore, it can be argued that trickle-down economics isn't the best method to combat rising inequality. Firstly, by cutting taxes, the rich are incentivised to save their money in tax havens like the UAE. Statistics show that reducing taxes does not increase economic growth in the long run; there was negative income growth in 1991, 10 years after the tax cuts in the USA. If there is no growth (or growth is unsustainable), inequality will rise; aggregate demand will decrease during times of stagnating growth which leads to greater unemployment. Marginal productivity theory suggests that a higher income is reflective of a larger social contribution to society. But trickle-down economics doesn't advocate this, because the incomes of the rich increase through no contributions (to society) of their own. During the Clinton and Bush administrations, tax rates on capital gains dropped to 15%. Capital gains consists of 57% of the income of the top 400 earners in the USA. By lowering the tax on capital gains the rich were essentially gifted an extra source of income. The effects of this tax reduction were recorded in 2007 and 2008 where the top 400 gained on average \$75 million each whilst tax revenues decreased by \$30 million each over the 2 years. Naturally lower income households feel the brunt of this tax reduction. Due to decreasing revenues the government cuts spending on social welfare programmes and education which causes inequality to rise. The IMF stated that GDP growth decreases if the top twenty percentiles of income experience growth. Thus, it can be implied that inequality worsens with trickle-down economics, because the upper echelons become richer and the poor don't experience any income growth.

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There are arguments which suggest that a wealth tax isn't the best way to combat rising inequality. Increasing taxes which affects the rich mainly would result in many wealthy individuals migrating to other countries with lower tax rates such as the UAE and Monaco. A wealth tax would severely affect the home economy as investment would decrease. A lack of growth widens the inequality gap, because there would be more unemployment. Rising unemployment only worsens inequality as the economy falls into a vicious circle, where consumption and investment keep decreasing. A wealth tax would also cause a greater social division between the rich and poor; the rich would feel as if the rewards of their hard work is being appropriated by the lower classes. Increasing taxes has a negative effect on government revenue; firms find methods to cut expenditure normally by laying off workers - and they end up paying less to the government. The Laffer curve suggests that revenue for the government would decrease if taxes are greater than the optimal tax rate. If government revenue decreases, the poor would be affected most harshly; social protection programmes and public investment in sectors such as education would decrease. Inequality worsens because the poor won't have good enough grades for high skilled jobs due to an inadequate education, whilst the rich - who have access to better quality resources - become more skilled and isolate themselves from the poor. A wealth tax would undermine society and cause inequality to actually rise, because the rich don't want to be forced to surrender their wealth. The wealth and income divide rises www.www.disproportionately with a wealth tax. I believe that a wealth tax wouldn't combat inequality as well as a UBI, because it doesn't help the poor but instead makes the rich poorer – demand drops, and economics instability rises.

Wealth inequality has been worsening since the turn of the millennium; as of now the top 1% of income earners own more than 50% of the world's wealth. Thomas Piketty says that if the 'rate of return on capital is greater than economic growth' then there will be wealth concentration at the top of society. We must acknowledge that a wealth tax could have an effect on rising inequality. Key advantages of a wealth tax include the fact that the rich would be able to afford it. Theoretically, it should allow median and lower income households to consume more (thus helping aggregate demand and growth) assuming that the government taxes them less with progressive tax policies using the revenue they gain from the wealth tax. However, in practise this isn't viable. The rich, with their extremely large influence in politics, are able to influence the government and central bank by funding political campaigns. This allows the rich to use their influence to promote policies that are in the best of interest of the rich. These policies are definitely not considerate of the poor, who have less political influence in comparison; they suffer at the hands of the rich. A UBI would be run independently, isolating it from greed and influence of the rich. A UBI is better than a wealth tax, because it takes political corruption out of the way, which means the rich can't alter it to suit themselves.

'The difficulty lies, not in the new ideas, but in escaping from the old ones,' John Maynard Keynes. We mustn't accept our current situation; we must actively try to combat inequality. A UBI scheme, I believe, would be the most effective in combating inequality, because it has a proven track record in pilot schemes ranging from the Mincome experiment in Canada (1970's) to the recent pilot schemes in Finland. The problem with the other methods is that they are too risky; there is a greater chance of economic instability which would worsen inequality. They also don't tackle inequality as directly as the UBI. For example, a global wealth tax forces the wealthy to bypass the tax by saving their wealth elsewhere. Inequality is actually forced to rise with the other schemes, because they give a chance to the rich to escape their responsibility towards society - something which a UBI does not allow. The UBI promotes an egalitarian society; other methods still allow inequality to exist which fundamentally undermines the poor. To combat inequality, we must, however, embrace the fact that in the near future we will have to work less due to the rising use of automation/technology in jobs - Keynes predicted that in 2030 there would be 15 hour working weeks! But a UBI actually favours this outlook. It gives the precariat more time for leisure and unpaid work. History tells us that a UBI scheme would be best to combat rising inequality - whether it be Thomas More in his book Utopia or human rights activist Martin Luther King JR - they all believed in a basic income to combat rising inequality. For those not as sceptical, like me, a UBI provides a sense of justice to the poor - they don't deserve to live in fear of economic instability despite working in the most physically strenuous sectors. It's imperative that we stop the rich from getting richer whilst the poor don't experience similar prosperity. The time for a UBI is now. 'Nothing is more powerful than an idea whose time has come' Victor Hugo.

Can bioengineering and biomimicry be the solution to our medical and engineering problems?

Edward Ferguson

Through the process known as natural selection, nature has designed its best, most efficient solutions available to adapt to the various environments imposed upon it. Biomimicry is the idea of learning from natural design and trying to reapply it to solve our own engineering problems, often in the field of bioengineering. Not only does it allow us to discover ways of being more sustainable with our choice of materials, but it also shows us how we can improve upon current designs by looking at the problem from a different angle and providing alternative solutions.

One excellent example of this is the inspiration provided by the technology of the Bombardier beetle's spray mechanism to improve spray technology for the likes of inhalers, fire extinguishers and fuel injection systems. Bombardier beetles have a very efficient mechanism in the form of the muscle pumped defensive spray which ejects boiling liquid towards the beetle's aggressor. The mechanism works by having a gland containing hydrogen peroxide and a gland containing which are both pushed into the abdominal chamber by the constriction of the muscle surrounding each gland and are allowed to enter through the inlet valve and mix¹. The combination of the two chemicals causes an exothermic reaction catalysed by the oxidative enzymes, such as peroxidases, secreted from the thick walls of the chamber. The reaction produces p-benzoquinones which are compounds well known for their irritant properties². The heat generated from the reaction significantly increases the pressure inside the chamber until the

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http://www.nbcnews.com/id/24637825/ns/techn ology_and_science-innovation/t/beetles-toxicblasts-trigger-innovation/#.Wy-LqadKi00

² http://prince.org/msg/105/199864?pr

critical pressure is reached and the exit valve is forced open causing a sudden decrease in pressure; the solution evaporates as it is at a higher temperature than its vaporisation point (also known as "flash evaporation"). Ejection and closing the valve triggers the repeating of the process. The beetle is then able to eject the spray at a rate of about 500 times per second³.

Engineers are attempting to use these mechanisms in spray technology since it does not require a propellent and is less complex than current spring-powered mechanisms. Also, the beetle's technology allows a finer spray that is distributed for evenly over a greater distance. This will allow fire extinguishers to spray further and more evenly, inhalers to produce a spray consisting of finer droplets which aid drug delivery as well as significantly improving the efficiency of fuel injection systems in both cars and planes. The Swedish company, Biomimetic 3000 Ltd has already begun research and development in this area and are looking to bring the technology into the various markets on a large scale.

Biomimicry and bioengineering are at the cutting edge of engineering and medicine-related research. For my ILA I researched some of the developments in this fascinating area and conducted some research of my own in an attempt to mimic and perhaps improve on nature. You will see if I succeeded!

Engineering Biocompatibility

I would say that the key to Bioengineering becoming the future of engineering problem solving, particularly in combination with medicine, is making sure that materials being used are biocompatible with the environment they are going to be used in. Biocompatibility is the is the ability of a material to interact with the host

³ Aneshansley and Eisner, 1969; Dean, et al., 1990; Eisner, et al., 2000.

without any undesirable effects such as triggering a toxic or immune response (mainly just in humans) or leaching toxic particles into the surrounding area when coming into contact with the environment in which the device or structure is being put (for example the human body). Biocompatibility gives devices a much greater lifespan because they do not decompose and need replacement or negatively interact with their host environment and thereby damage the host. This idea was made clear when pilots during WW2 who had had their canopies smashed and had bits of the Perspex in their eyes did not have any side effects from having this material in contact with their bodily fluids. Perspex (or PMMA) was discovered to be "extremely biocompatible, resistant to long exposure to temperatures, chemistry and cell action of human tissue"4. This characteristic of Perspex was noticed by Harold Ridley, an Ophthalmologist, and he went on to use Perspex for a whole manner of intraocular products such as contact lenses.

An understanding of biocompatibility is a necessity in areas including joint replacement and bone fracture repair because materials that are at odds with the patient's bodily fluids with most likely cause pain and decrease mobility. When attempting to replace a body part, such as a section of bone, you must try and replace it with a material that not only has the most similar mechanical properties but also has the ability to interact with the live cells around it so that it becomes an integrated part of the patient's biology. For example, a synthetic replacement for bone would have to be very capable of experiencing large loads without deformation and would have to take into account the fact that bone is a living tissue full of living cells. This means that as well as interacting with the other cells around it in the body, it also undergoes a remodelling process where the bone replaces itself completely over the course of every 7 years. It is for this reason that previous solutions to larger or unstable bone injuries, such as mineral bone graft (only the mineral components of bone), have not been long term solutions because they only provide a structural repair to the damaged bone which means that the patient's bone biology is compromised.

The current most common choice for bone replacement is an autograft. An autograft is a section of bone taken from another bone in the body of the patient which is used in the repair of the damaged bone tissue. This sort of replacement is beneficial to the patient because the autograft has the same constitution as the bone surrounding it meaning that it will promote cell-mediated remodelling. The only issue with this is that there is a limited supply of spare bone material in a person's body and an extra surgery is required to source the autograft, putting the patient at more risk than is necessary. At QMUL, research by Dr Karen Hing has built upon their previous research conducted where the "graft chemistry was enhanced by introducing silicate into hydroxyapatite", a substitute material that phosphate, calcium and contains whose chemical structure is very similar to that of natural bone. "Silicon distribution in the body is linked to its biological activity, especially in terms of the functions of connective tissues, and in particular bones"5. Her more recent research builds on the discovery that the "combination of optimised chemistry and pore structure was better at guiding cells to differentiate into cells that produce bone tissue". Inductigraft, the ceramic material created by Dr Hing has pores at both the micro scale in the struts of the structure through which ions, proteins and nutrients can be exchanged, as well as the larger scale in which the bone grows. Inductigraft has 10% of bone graft market share in the USA and has already been used by 200,000 people globally⁷. This ceramic is an impressive feat of engineering combining engineering with human biology and chemistry to produce a product that clearly improves the quality of life of those who require it. The success of Dr Hing's research is down to her ability to recognise that our body's natural method of bone regeneration is still the best method to use when the bone cannot regenerate in its usual way. Engineering a ceramic material that can encourage and promote this behaviour of a patient's bone is the ideal solution to the problem of severe fractures and bone trauma.

Potentially in the future of this area more focus could be put on the understanding of why the Inductigraft is such a good synthetic bone material and looking at the possibility of promoting bone regeneration without the requirement of surgery to input the ceramic. For example, the proteins, ions and other "signals" that cause bone regeneration naturally could be harnessed and used in an injectable fluid so that

⁴ http://blog.grantadesign.com/all-aboutmaterials/pmma/

⁵ https://doi.org/10.1016/j.pnsc.2017.08.009

https://www.qmul.ac.uk/media/news/items/se/16 8709.html

⁷ https://youtu.be/Hu_rlVUuPdQ



there could be even fewer risks associated with recovery from bone fractures.

Bone loading and its various applications

Bone is a very intelligent material because it will react to the mechanical forces it experiences by reinforcing the bone structure at high stress areas but more importantly, the bone "in a healthy person or animal will adapt to the loads under which it is placed"8 - Wolff's Law. It is the cancellous bone (softer, more porous and flexible central section of bone) that adjusts its structure to accommodate the mechanical loading it experiences. The cancellous bone consists of lots and lots of trabeculae (small strut-like tissue element that provides support or anchoring within a tissue which in this case is bone) which are optimally positioned to resisting loading during activities such as running and jumping. Red bone marrow fills in the gaps between the trabeculae. As each trabecula is aligned to resist loading from the cancellous bone, the cortical bone (harder, outer layer) follows suit and eventually the bone has adapted to effectively resist external loading.

This bone activity is something that is of particular interest to bone physiologists because they need to understand how implants, particularly total joint replacements will change the loading on the patient's bones compared to before the inputting of the device. They will have to analyse the patient's trabecula bone mechanical properties such as fatigue and Young modulus, in order to create implants that will be effective in conjunction with the person's own physiology. Cancellous bone is normally 75%-95% porous meaning that it is trading off strength for weight, however, the trabeculae are orientated in such a "direction at which mechanical stiffness and strength are greatest". Trabecular bone is very anisotropic, meaning that it does not having the same mechanical strength in all direction, in fact it can only deal with high loading in roughly a single direction.9

Trabecular bone does not have a regimented structure making it heterogenous and therefore difficult to give general mechanical properties to, especially as ageing of the tissue, small fractures can have a big effect. In general, the porosity of the trabecular bone is inversely proportional to the strength and modulus of the bone.¹⁰

This idea of bone structure being adapted to deal with loading and absorbing the stress of impacts is also exhibited in the beak and skull structure of woodpeckers. In fact, the allometry of all birds allows them to experience high loads compared to their bone mass because their bone structure gives them a very high specific mass and a resistance to buckling due the combination stiff outer shell with a spongy, flexible trabecular centre. The inner network of trabeculae shifts mass away from the neutral axis, which ultimately increases the resistance to buckling.¹¹ The density of trabeculae throughout the woodpecker's head is significantly greater than the rest of its body and unlike a lot of birds, whose trabeculae are rod-like, the woodpecker has ticker plate-like trabeculae which are so shaped in order to reduce the stress on the bird's brain.¹² This gives the woodpecker a significantly stronger bone structure compared to other birds (average ultimate strength of 6.38MPa, compared to the lark's 0.55MPa)¹³.

Engineering applications of the adaptations of the woodpecker's bone structure include the protection of the black boxes kept in aircraft. The protective layers of the back boxes include hard titanium or steel, an elastomeric element to disperse the vibrations and finally a porous layer of glass microspheres to mimic the trabecular bone structure. The black boxes are able to withstand up to 60,000 Gs. Additionally, this technology is becoming widely used in helmets; the Kranium Paper Helmet created by Anirudha Surabhi has used the idea of the trabecular structure in woodpeckers (actually uses hexagonal structure with cardboard for

⁸ Anahad O'Connor (October 18, 2010). "The Claim: After Being Broken, Bones Can Become Even Stronger". New York Times.

⁹ Meyers, M. A.; Chen, P.-Y. (2014). *Biological Materials Science*. Cambridge: Cambridge University Press.

¹⁰ Hayes, Wilson C.; Keaveny, Tony M. (1993). *Bone: A Treatise* (PDF) (7 ed.). CRC Press. pp. 285–344.

¹¹ Meyers, M. A.; Chen, P.-Y. (2014). *Biological Materials Science*. Cambridge: Cambridge University Press. pp. 504–506.

¹² Wang, L.; Zhang, H.; Fan, Y. (2011). "Comparative study of the mechanical properties, micro-structure, and composition of the cranial and beak bones of the great spotted woodpecker and the lark bird".

¹³ Wang, Lizheng; et al. (2013). "Effect of Microstructure of Spongy Bone in Different Parts of Woodpecker's Skull on Resistance to Impact Injury". *Journal of Nanomaterials*. **2013**: Article No. 17.

manufacturing reasons) to be able to three times more force whilst being 1500 times lighter than standard polystyrene helmets¹⁴.

The mechanical properties that this technology supplies have the potential to be sources of aid to structural engineers trying to facilitate the designs of pioneering architects as well. The high specific strength gives rise to the possibility of having buildings that require smaller foundations because there is not such a need for dense building materials to be able to take the loads of a multistorey building. Furthermore, the trabeculae's ability to minimise the chance of buckling along the neutral axis could allow bridge designs to require fewer support structures or even allow far areater loads on the bridge without failure. In the mechanical department, vehicles can be made lighter and still able to accommodate larger impacts, progress is already being made into the redesigning of helmets which will reduce the risks of mental health problems throughout the population.

Another fascinating use of the loading properties of "bone" or bone-type material as a potential construction material. Michelle Oyen at Cambridge University points out that around a tenth of the carbon emissions is the world come from production of steel and concrete. She is suggesting that creating an artificial bone, small samples of which she has made in her lab (see this video where they use robots to make the bone¹⁵), could be used as low-carbon building materials if produced on a commercial scale¹⁶.

Engineering Intelligent Biomaterials

Engineering an intelligent biomaterial might not be as difficult as you think. Simply changing the molecular structure of a compound could result in huge changes to the characteristics of the material. Take graphite as an example, a brittle allotrope of carbon that consists of layers of carbon atoms known as graphene joined to each other by weak forces. A simple change to the bonds between the carbon molecules, namely

 ¹⁶ Would you live in a city made of bone? | University of Cambridge
https://www.cam.ac.uk/research/features/wouldyou-live-in-a-city-made-of-bone 23 Jun 2016 covalent bonds in three dimensions, gives rise to the strongest natural material known to man. This shows just how important the molecular structure of a material is on its mechanical properties as a whole.

The physics behind the flexibility of red blood cells was recently discovered using super-resolution microscopy; the surface of the RBCs is made up of a 2D structure of interlinking triangles of a protein called spectrin which is like a spring, allowing the cell to bend and flex. The geodesic structure is the only structural support in the of the red blood cell which explains why it is so flexible.

Another kind of intelligent biomaterial is that on the feet of a gecko. It is a well-known fact that geckos can walk on pretty much anything at any angle, but the reason why is not so famous. The surface of a gecko's foot is made up of half a million setae (tiny hairs) that protrude from the epidermis of the gecko (which is made of betakeratin) and increase the Van der Waals forces acting between the foot and the surface.¹⁷



The nanostructure of the gecko's foot shows the spatulae (of which there are roughly 1,000 per seta) on the end of the setae (of which there are roughly 14,000 per foot). The fact that the size of the spatulae is on the nanoscale is very important because through research it was found that small variations in the shape of the contact surface severely affected the adhesive properties of the contact. The method of adhesion used by Geckos is 'dry' in that it does not secrete anything of its own to help stick to the surface. In fact, it makes use of the thin layer of water present on every terrestrial surface. The water increases the strength

¹⁷ Autumn, K.; Setti, M.; Liang, Y. A.; Peattie, A. M.; Hansen, W. R.; Sponberg, S.; Kenny, T. W.; Fearing, R.; Israelachvili, J. N.; Full, R. J. (2002). "Evidence for Van Der Waals adhesion in gecko setae". *PNAS*. **99** (19): 12252–12256.

¹⁴ Felman, Rachael (March 22, 2013). "Bird-Brained Bike Helmet Coming This Summer". *Popular*

Mechanics.

https://www.cam.ac.uk/research/news/growingbones-with-lego

of the Van der Waals forces and higher humidity means a stronger attraction ¹⁸.

At the Max Planck Institute for Metals Research in Stuttgart, Germany, researchers have used this technology to develop a biomimetic material that can stick to smooth walls without any adhesives. This technology has a vast amount of possibilities for applications in the future and is already being used on production lines for glass components, but it is still in its early stages and needs a fair amount of work still done on that area¹⁹.

In another case of take a leaf out of nature's book, Gecko Biomedical have made an adhesive that works in wet conditions, is blood resistant and is activated by UV light. The technology is taken from sandcastle worms which secrete sticky protein-based glue granules that join together and set in thirty seconds in cold sea water. "Gecko Biomedical has packaged the glue into injectable nanoparticles for delivery in minimally invasive surgeries. Once they are exposed to positively charged particles, the nanoparticles fuse into a functional glue."20 Once in place, exposure to UV light will set the glue in five seconds. This glue is viscous, flexible and biocompatible which allows it to flex and move with soft tissues and it does not have any harmful effects on the body. The adhesive has also been designed to degrade over time so that eventually the tissue has completely healed. The team at Gecko Biomedical are now looking at using the glue to act as a drug delivery system so that antibiotics can be released at the glue degrades. This sort of technology is extremely useful for surgeons, especially in keyhole surgeries where it is very hard to join tissue in such a confined space.

INVESTIGATING THE COMPETENCY OF RUBBER AND COLLAGEN AS SYNTHETIC ARTERIAL TISSUE

Having looked at how biomimicry was being used to generate improved materials for implants, I thought I would look at the bioengineering of artificial replacement arteries, in particular as my brother had to have a section of his aorta replaced with a Dacron tube to prevent an aneurysm developing and I am aware that it does not behave in the way normal aortic tissue would

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https://www.nanowerk.com/spotlight/spotid=118

https://asknature.org/idea/surgical-glueadheres-wet-and-sensitive-tissues/#.W1dr39JKi00 in response to blood pressure stresses. I wanted to assess the suitability of different types of materials as synthetic arterial walls which might would be, so I designed an investigation to see what I could find out, within the limits of the apparatus available to me in my school lab.

Background: Currently, most arterial replacements are made of fairly resistant plastics (e.g. Dacron grafts²¹) which do not respond to blood pressure in the same way that arteries do (the thick walls of arteries absorb some of the pulses of pressure produced by the heart's pumping action and smooth out the flow, whereas the plastic tubes do not allow this effect to such a great extent). Replacement sections are often needed because the patient's arteries have ballooned into aneurysms which could burst and cause the patient to die suddenly. In such a scenario, a strong plastic tube is obviously better than a weak natural tissue wall, but it would be good to be able to produce something that mimics the natural aorta wall. This was the motivation for my investigation.

Design of the investigation: I designed an experiment to investigate the effect of force on the rate of extension of a material. I chose this because it was similar to the way in which blood flow at different blood pressures acts, exerting different magnitudes of force on the artery walls and so stretching the tissue at different rates. The materials tested were rubber bands and collagen in order to try to replicate the materials that make up arterial walls. Elastin and collagen are the major extracellular matrix (ECM) constituents in large elastic arteries²². To replicate this, I used rubber elastic bands instead of elastin as elastin is described as "a rubber-like substance"23. I also examined the properties of collagen which I managed to get hold of in the form of material used to make sausage casings. I wanted to compare the material properties of synthetic materials compared to actual arterial

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¹⁸ See images on next page

²¹ https://goo.gl/images/j37r6W

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5 201077/

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4 996762/

²³ See page 91 under heading "The properties of blood vessel walls" in the book: The Mechanics of the Circulation (2nd edition) by Caro, Pedley, Schroter and Seed, Cambridge university Press (2012)

components in order to investigate potential aortic replacement materials.

I chose to test the rate of extension when different forces were applied longitudinally, rather than extension alone, in order to mimic the effect of blood pressure on the arteries and how the blood is forced against the walls of the arteries at varying speeds, which will cause variation in the stretching of the arterial walls. Also, for reasons of complexity, I only stretched the materials in a linear direction because I did not have the equipment to stretch the materials in multiple direction and measure each force acting on it even though it would have been more representative of the forces experienced by actual artery tissue.

My null hypothesis was that the change in force exerted on the test material would not affect the rate of extension of the material (the speed at which it extends).

Apparatus: I set up my apparatus of: a motor with pulley system; string; 3 rubber bands; beef collagen (sausage casings); force sensor; metre rule; 2 G-clamps; metal stand; 2-6V power source; data-logger. The apparatus was set up so that the motor with a pulley system who be supplied with a certain voltage and the string attached to the rubber band/collagen would be attached to the pulley and be pulled away from the force sensor (which was fixed). See a diagram of the apparatus I used below:



Method: I recorded the force exerted (dependent on the voltage supplied to the motor) on the rubber bands and collagen at each voltage and formed graphs such as the one below for the first elastic band used.

<u>Results:</u>



Force/N vs. Time/ms for rubber band 1, 5V

The graphs above and below represent the response of the elastic and collagen I tested in relation to increased forces applied to them (specific results were included in my original ILA but not here, to save space in this shortened version).

The Youngs Modulus values I found for the elastic and collagen I tested were: E_E (elastic band) 0.00685 MPa and E_C (collagen casings) 0.0823 MPa.



Force/N vs. Time/ms for collagen, 2V

Comments on results: The data I collected from my experiments showed results which contradicted my null hypothesis that the force has no effect on the rate of extension; in my results, for each increase in force there is an increase in speed/rate of extension.

Discussion: I researched the material properties of human arterial tissue and some of its constituent elements and identified these as the Youngs modulus value for actual arterial tissue and its constituent elements²⁴: E_A (effective Young modulus for arteries) = 0.384 MPa; EE (elastin) = 0.3 MPa; E_C (collagen) = 200 MPa ²⁵. Although being very different to the Young's modulus values I found in my experiment for my potential alternatives (EE (elastic band) 0.00685 MPa and $E_{\rm C}$ (collagen casings) 0.0823 MPa) they do show a similarity in that the $E_{\rm c}$ was much greater than the E_E although not to such a great extent (the Youngs modulus ratio was approximately 666:1 in the in vivo tissue and only 12:1 in my experiments). This is likely to be due to the differences between elastic and elastin and the accuracy of the equipment used and the different conditions in which the testing was occurring).

 ²⁴ http://circ.ahajournals.org/content/94/12/3263
²⁵

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4 723153/



Nevertheless, these results suggest that a combination of the rubber band elastic material and the beef collagen (sausage casings) could have similar mechanical properties to arterial tissue, however the maximum pressure it could experience would be much less than real arterial tissue, so it would not be useful in practice. This difference in maximum capacity (and Youngs modulus) of an artery is understandable because arterial tissue contains other elements, including smooth muscle tissue (which has a Youngs modulus similar to elastin²⁶) whose function is to relax or constrict the artery, which will affect the forces it experiences from the blood flowing through it.

As well as the addition of muscle fibres, we must consider the differences in how the materials combine to make arterial tissue compared to the individual tissues alone and how the way they respond to fluid forces changes. The tunica media (middle layer) of arteries contains smooth muscle cells that are embedded in a matrix of elastin fibres, collagen fibres as well as aqueous ground substance²⁷ matrix containing proteoglycans²⁸. The tunica adventitia (outer layer) is made up of a dense network of collagen fibres. Elastin fibres join together to form concentric elastic lamellae with fibres connecting the lamellae. Each of the elastic lamellae alternates with a layer of smooth muscle cells and collagen fibres to form a lamellar unit the functional part of the of the artery wall. In addition, things like the orientation of the fibres influence the load-bearing ability of the tissue.

Collagen is much stiffer than elastin. The two substances work well together in arteries. Elastin is stretchy and can stretch to twice its unstretched length but the stress value at which it breaks is only 5 % of that of collagen which itself is much less "stretchy". Thus, the two work together, the elastin to take up the initial pulse and the collagen kicks in to prevent bursting when the stretch is getting to its maximum.

²⁸ https://en.wikipedia.org/wiki/Proteoglycan - A compound of a "core protein" with one or more covalently attached glycosaminoglycan chains.

The combination of the two main materials, elastin and collagen, gives rise to the non-linear behaviour of arterial tissue. The elastin controls the majority of the vessel's reaction to the forces from the blood until the collagen fibres are stretched enough that the molecules line up to their full extent, at which point the overall stiffness of the artery significantly increases because the collagen fibres have been engaged. This behaviour is "analogous to that of a balloon being blown up inside a string bag. It is relatively easy to start with but extremely difficult once the bag is taught."29 The graph below illustrates my point.



It is interesting to note that in dogs' arteries, where arteries are under extreme stress, for example very close to the heart in the intrathoracic aorta, the ratio of elastin to collagen is about 1.5, while in arteries under less pressure the ratio is about 0.5, and in veins, where pressure is at the lowest, around 0.3³⁰.

Conclusion: In conclusion, despite showing signs of having the appropriate mechanical characteristics to be a suitable replacement for arterial tissue, rubber band elastic and collagen would not be a good choice for making a section of synthetic artery; they cannot cope with the same scale of stress as an actual artery can because they do not have the complex and well-

 ²⁶ The Mechanics of the Circulation (2nd edition) by Caro, Pedley, Schroter and Seed, Cambridge university Press (2012)

http://medcell.med.yale.edu/histology/connective _tissue_lab.php - An aqueous gel of glycoproteins and proteoglycans that occupies the space between cellular and fibrillar elements of the connective tissue. It has a gel-like viscous consistency and is polyanionic.

²⁹ See page 96 under heading "The properties of blood vessel walls" in the book: The Mechanics of the Circulation (2nd edition) by Caro, Pedley, Schroter and Seed, Cambridge university Press (2012)

³⁰ The Mechanics of the Circulation (2nd edition) by Caro, Pedley, Schroter and Seed, Cambridge university Press (2012)

evolved structure of an artery and rubber bands, particular, have a tendency to tire. This means that not only would the replacement artery be too weak but it's elastic capacity would reduce with time until the rubber snapped with ease.

This of course is not a sustainable solution for people with arterial issues and would most likely cause more damage to the patient than potential benefit. Therefore, I think that Dacron, the present solution, is a good solution for the time being because it can cope with high pressure flow and last much longer than any natural alternatives available right now.

OVERALL CONCLUSION OF MY INDEPENDENT LEARNING ASSESSMENT

My Independent Learning Assessment has focussed on the natural world as a resource for solving our engineering problems. The combination of classical engineering of big, physical structures and mechanisms, and more modern engineering on the nanoscale, with a greater knowledge of the nuances of the environment in which we design, will be the source of the best solutions to our engineering and medical problems.

The independent investigations into materials suitable to replace a biological tissue which I conducted in the lab and have described above have made me understand that it is not just the mechanical properties of a material that are key; technology biomimetic must also he biocompatible with its host so that the body does not reject the substitute. In the case of an artery, which I was investigating, a non-compatible material could cause blood clots, an immune response or even leach particles into the body over time and poison the patient. Current research is looking at triggering natural cell growth around a scaffold³¹ to replace extended sections of arteries.

Nature produces the most well adapted design and materials currently; we can only hope that bioengineering will be able to catch up with it soon. As Einstein is quoted as saying "Look deep into Nature, and then you will understand everything better".



https://www.sciencedirect.com/science/article/ pii /S0022522315017936

The modern-day sugar epidemic and its negative health effects; why has its use been recently limited?

Gabriel Johnson

Sugar is the blanket term used to describe soluble, short-chain carbohydrates, namely table sugar (sucrose), which I shall henceforth refer to as sugar; disaccharide this α comprisina two monosaccharides called glucose and fructose. Sucrose accounts for a growing proportion of our daily calorie intake, with the National Diet & Nutrition Survey of 2008 showing that UK adolescents acquire 15% of their daily calories from it. The vast augmentation in sugar consumption over the past half a dozen decades has led to the increase in the rates of obesity, and diseases including diabetes mellitus type 2 and coronary heart disease. In this essay I intend to delve deeper into some of these diseases, exactly how sugar is related to them, and ultimately why there was a sugar tax recently passed in April 2018.

Firstly, one must better understand why exactly sugar is causing so many health problems, and all the issues essentially stem from one particular trait of sugar: it is addictive. In fact, sugar is so addictive, it stimulates the same reward centres of the brain as cocaine and morphine. A study carried out in 2007 proved that rats showed a preference to sugar dissolved in water over cocaine, and showed withdrawal symptoms (including, but not limited to loss of appetite and weight loss) when the sugar water was later removed from their cages. With these findings in mind, one can compare sugar addiction to that of psychoactive drugs: the consumption of sugar triggers the release of dopamine across neuronal synapses. Dopamine is a neurotransmitter which produces a euphoric sensation as a means of rewarding the individual for specific behaviour, often to incentivise behaviours which guarantee the individual's survival. This feeling is very short-term, which leads the individual to repeatedly pursue it (ensuring a higher chance of survival), with each time the brain requiring more of the substance to achieve the same high. In the case of sugar, dopamine is released from the synaptic vesicles since sugar contains the monosaccharide glucose: a necessary metabolite in aerobic respiration, releasing energy in the form of ATP - sugar therefore acts as a short-term energy source. However, in modern day society, sugar is not only used as an energy source, taking the form of energy drinks, but primarily for its flavour, particularly in soft drinks and sweets. Following repeated

consumption of, for example, a sugary fizzy drink, the



consumer will not only keep coming back for more, but will drink it in larger quantities to satisfy their sweet tooth; much like with tobacco or morphine, the individual develops a dependency on sugar. When combined with the fact that sugary products are legal, cheap and readily available, the individual becomes trapped in a downward spiral of their health, often without even realising it.

Perhaps the longest disputed health issue regarding sugar is that of coronary heart disease, with a large increase in the rates of this disease starting in the 1950s. As a result, the Sugar Research Foundation conducted a coronary heart disease research project in 1965, sponsored by the food industry; it demonised saturated fat and cholesterol, stating that they were the main cause of coronary heart disease, completely disregarding sugar. Seven years later, however, John Yudkin, a UK food and health medical representative, highlighted the clear bias of the study in his book Pure, White and Deadly, underlining the exaggerated role of dietary fat in causing heart disease, which was done to protect the growing sugar industry. Yudkin's book, supported by many other studies in the same decade, demonstrated that sucrose consumption resulted in hypertriglyceridemia (elevated triglyceride levels in the blood), more so than starch (a polymer of glucose molecules). It was concluded that this was because of the presence of fructose (the key ingredient of high fructose corn syrup: an industrial sweetener which rapidly became widely used) in sucrose, but not in starch. Fructose, unlike glucose which is respired in living cells all over the body, is metabolised exclusively in the liver via fructolysis. Of course, the science behind it is not so simple; given

the circumstances, fructose can be metabolised in many ways, but in the presence of excess fructose, de novo lipogenesis (DNL) occurs. DNL is the metabolic process through which the liver converts excess monosaccharides into fat, and since a much larger proportion of fructose is metabolised in the liver than glucose, fructose has a higher tendency to stimulate DNL. The fructose is converted to dihydroxyacetone phosphate and glyceraldehyde-3-phosphate, the latter of which rests in equilibrium with glycerol-3-phosphate - this is easily converted into alvcerol (the backbone molecule of triglycerides). In the meantime, the majority of the carbon atoms in fructose's pentose ring are metabolised into pyruvate and consequently into acetyl-CoA, having undergone the link reaction. Acetyl-CoA is the primary compound from which fatty acids are synthesised, and three of these fatty acid molecules combine with a single glycerol molecule to form a triglyceride. Therefore, DNL alone produces the



leads to hypertriglyceridemia. These raised levels of triglycerides in the blood increase the likelihood of atherosclerosis: the build-up of plaque in arteries, including the coronary arteries of the heart. This plaque builds up, hardening and narrowing the coronary arteries, reducing the blood, thus oxygen supply to the heart, the results of which include heart attacks and shortness of breath, namely during physical exertion. Evidently, none of these are desirable, for the individual or the NHS (coronary heart disease was estimated to have directly costed the NHS £1.8 billion in 2009), hence contributing towards the limitations placed on sugar.

Type 2 diabetes mellitus (T2DM) is another wellknown consequence of excessive sugar ingestion, with an estimated 400 million diagnosed people with the disease in 2015. It is a result of either the pancreas' β -cells producing insufficient insulin to cope with blood glucose levels, or by cells' insulin receptors denaturing and no longer being complementary to the hormone's tertiary structure, thus preventing the facilitated diffusion of glucose across the phospholipid bilayer of the cell membrane. Sugar can cause T2DM in one of two ways, because, as mentioned earlier in this essay, sucrose is composed of two monosaccharides: glucose and fructose. Foods and drinks which quickly raise blood glucose levels (that is by having a high glucose or sucrose content) are described as having a high-glycaemic index; due to raised blood glucose levels, more advanced glycation end products (AGEs) – which are glycoproteins or alycolipids - are produced. These AGEs lead to an increased volume of fat tissue and inflammation: the fat tissue (especially expanded fat tissue) in turn produces a substance called proinflammatory cytokine TNF-a, which causes insulin resistance, thus T2DM. As for fructose, excessive consumption results in hypertriglyceridemia due to increased de novo lipogenesis, as mentioned earlier - the increased number of triglycerides in the bloodstream causes the body to lay on ectopic and adipose fat, also releasing proinflammatory cytokine TNF- a. Among the many long-term risks of T2DM are nerve damage (resulting in a higher likelihood of requiring an amputation), strokes, and ketoacidosis, where the body uses other hormones to turn fat into energy due to the lack of glucose in cells - this causes a build-up of toxic ketone acids. With an estimated 175 million people globally having undiagnosed T2DM, this is a major public health concern, especially since the early symptoms - including increased thirst, urination and fatigue - can easily go unnoticed. Therefore, in the UK, introducing the sugar tax is the first step towards reducing the colossal financial strain that is placed on the NHS because of T2DM, with approximately 10% of the NHS budget (around £14 billion) dedicated to treating it annually.

Lastly, there has been substantial research into the link between type 2 diabetes and dementia, specifically Alzheimer's, leading some to coin the phrase "type 3 diabetes". With the rates of T2DM on the rise, this is of upmost concern, with the rapidly increasing ageing population becoming ever the more vulnerable. Alzheimer's is little

understood, but is essentially cell death and tissue loss of the brain – the cause of this is postulated to be beta-amyloid proteins. These proteins clump together in the brain, blocking the



diffusion of neurotransmitters across synapses, and potentially stimulating the humoral immune response to destroy neurones. Given all this information, how does T2DM relate to Alzheimer's? It is speculated that insulin-degrading enzyme (IDE), catalysing the breakdown of both insulin and beta-amyloid proteins, is the source. In T2DM patients, IDE levels are low because of a lack of insulin (since β -cells are exhausted from

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overproducing insulin in response to insulin resistance), after all, why have lots of IDE when there is no insulin to be broken down? Therefore, there is not enough IDE to break down the growing beta-amyloid plaques. Furthermore, because of T2DM, the brain's cells are insulin-resistant, meaning they cannot absorb sufficient alucose for aerobic respiration - they begin dying, initiating the symptoms associated with Alzheimer's disease, such as disorientation and memory loss. Suzanne de la Monte, a neuropathologist, demonstrated this with rats, interfering with insulin levels in their brain which resulted in their brains displaying all the common signs of Alzheimer's - does this mean diabetes is the cause of the neurological disease? No, it does not, rather they are linked through the same root cause, with sugar playing a major part in both. Note that the content of this paragraph is purely speculation with scientific evidence to support it; much research is still taking place regarding Alzheimer's, and, so far, we have only proved a correlation between insulin levels (thus T2DM and sugar overconsumption) and the brain's deterioration. Perhaps with greater research a causal link between sugar and Alzheimer's will be established.

To conclude, after decades of delusion, sugar (sucrose) has been unveiled as the main culprit of many of the greatest public health issues that our generation will have to face. A colossal industry has been built around sugar's addictive nature and sweet taste, with decades having passed before action was taken to limit its use because of corporate greed. With the diseases mentioned in this essay becoming all the more frequent, not placing limitations on sugar would be putting both the public's health at risk and more financial pressure on the NHS, so the recently introduced sugar tax is one of the many restrictions we can expect in the years to come.

Behind Shakespeare's Antonio and Venice

'The Merchant of Venice' is one of William Shakespeare's most controversial plays, touching upon anti-Semitism, gender inequality, comedy and tragedy; and is believed to have been written between 1596-1597. It is not known if Shakespeare ever visited Venice though his whereabouts are unknown between 1585 and 1592. These seven years are known as the 'lost years'. There is much debate regarding what he did during this period. As thirteen of his plays are either set in or bear reference to Italy, there is speculation that Shakespeare visited Venice, probably during the 'lost years'. If he did, it would have heavily influenced his writing of 'The Merchant of Venice'. In this essay, I want to establish two things:

The first is to examine what made Venice and its merchants of sufficient interest to Shakespeare to use as a backdrop to the play. (Act 1)

Secondly: To review the historical influences that may have provided the basis for the relationship between Antonio and Shylock. (Act 2)

By considering the historical evidence, I hope to gain a better understanding of 16th Century Venice and who the merchant, Antonio might have been.

Act 1: Antonio and Venice

(Why would Shakespeare choose Venice as a setting for his play?)

Scene 1: From salt trade to La Serenissima

Arguably, Venice was built upon the salt trade. Before Eastern spices became mainstream in European markets, salt was the easiest way to alter the taste of food and essential to preserve food since it dried out meat and fish, allowing Europeans to store food for longer; and avoid famine. Venice is built on a salt water lagoon and the abundance of marine life provided protein to the Venetian diet and gave Venetians their first viable export: salt. It was traded for grain and other essentials that Venice could not produce itself. John Julius

Harry Dennis

Norwich in 'A History of Venice", quotes the importance of the salt trade to early Venetians in a letter written by Roman statesman Cassiodorus in 523 AD:

"All your energies are spent on your salt-fields; in them lies your prosperity, and power to purchase things which you have not. For though there may be men who have little need of gold, none live who desire not salt".

Salt provided a constant income to the lagoon people, uniting them. The early Venetians sold salt collectively, to drive up its price and create increased mutual profits. Venice soon held a monopoly over the Veneto salt trade. By the end of the 13th Century, Venice had expanded from providing salt to the Po Valley and Tuscany, to Rome, Sicily, Crete and Cyprus, but Venice soon had rivals. The desire to dominate the salt trade led Venice to invade coastal areas of Croatia and Albania, to prevent the price of salt being driven down. Once occupied, these towns had their salt production closed, making Veneto the dominant producer in the Adriatic. The Venetian salt trade put Venice 'on the map' and supported its expansion into other trading markets. Some 1,000 years later, it would provide the foundations for Antonio's mercantile existence.



In a podcast entitled 'The Renaissance', Dr Gabriel Neher described the Black Death (arrived in Venice in January 1348) as "the great leveller". It killed rich and poor alike, tearing apart previous beliefs that those in higher authority were held in power by holy favour. Suddenly, anybody could be rich. This drive to get rich quick was aided by a



people vacuum left by the Black Death. Entrepreneurship grew in Venice and built a consumerist economy. Dr Neher proclaims this as the beginning of the Renaissance. Wealthy families were able to buy luxuries, creating an entirely new market. The suppliers were Venetian merchants which would have placed Antonio in an influential position in society.

Scene 2: The gateway between East and West

Of importance to Venice emerging as a trading superpower was its relationships with other nations. The Frankish invasion in 810 AD threatened the existence of Venice but the Frankish fleet was destroyed by Venetian sailors who knew precisely how to navigate the depths and shallows of their lagoon, the Franks simply got stuck in the mud. In 814, a treaty was agreed where Venice remained part of the Byzantine Empire, but paid annual tribute to the Franks, this was essential to Venetian success. It meant Venice was not under the direct rule of one empire but remained quasiindependent. Venice now sat at the centre of Europe, the Byzantines in the east, and the Franks in the west. This grey area of who Venice really belonged to was crucial to its independence, essentially it chose which side it sat on, depending on the situation. The most important factor in the treaty was a clause enabling Venetian trading vessels to utilise all Italian ports, and major ports of the Byzantine Empire. In Peter Ackroyd's words, "Venice became, predominantly, a city of merchants". Merchants like Antonio owed a great deal to this treaty since it paved the way for Venice's unique stance with the Near East.

Unlike other European powerhouses, Venice enjoyed a profitable alliance with the Ottoman Empire, trading through Ottoman lands; and further with the Middle and Far East. Venice became the only place to provide previously unseen merchandise to European markets. By the mid-16th Century, Spain, the United Provinces (Dutch Republic) and Portugal were competitors, but Venice had held sufficient monopoly until then, for it to dominate European trade.

Antonio's role and importance comes from business. Merchants from Venice travelled across Europe and beyond, bringing back cloth from England and silks and spices from the Far East and further afield, imported Eastern culture, architecture and food and commodities unseen in Europe. This fuelled the consumerist economy in Venice. The Venetian merchants were the clear winners. They were the gateway to Eastern markets and the only merchants permitted to travel there.

These goods brought back from the East added to the increased interest in Venice from other countries such as England during the late Middle Ages and the Renaissance. Writing during the early-16th Century, Canon Pietro Casola, described the scale of the mercantile scene with amazement:

"Indeed it seems as if all the world flocks here, and that human beings have concentrated there all their force for trading... who could count the many shops so well furnished that they almost seem warehouses, with so many cloths of every make – tapestry, brocades and hangings of every design, carpets of every sort, camlets of every colour and texture, silks of every kind; and so many warehouses full of spices, groceries and drugs, and so much beautiful wax! These things stupefy the beholder."

Venice never was self-sufficient, it relied on the profits from its merchants to buy essential resources. Good relations with the East were thus essential. By the end of 16th Century it was already established as the trading centre providing exotic goods.



Scene 3: Venetian politics and consumerism

Arguably the greatest change between the Middle Ages and the Renaissance in North-Western Europe was the move away from the Feudal System. Venice had no feudal system. It did not rely on a king or hereditary rule; the most powerful were the wealthiest. The wealth did not come from hereditary land ownership (since Venice is built on a lagoon) but from entrepreneurship. In his letter to the early Venetians, Cassiodorus describes the early class system by using the example of early Venetian housing:
"Among you there is no difference between rich and poor; your food is the same, your houses are all alike".

This lack of feudalism in Venice can be seen as a primitive system of Republicanism, which American scholars have credited as the ancestor of their own political system. In 'Venice Reconsidered', edited by John Jeffries Martin and Dennis Romano, they review the historiography of the Venetian political system and describe how American scholars view Venetian politics in the post-war period as "rather triumphalist". Without a feudal system a more fluid class system emerged, allowing Venice to be more progressive. People understood there was no limit to what they could achieve, it encouraged competition and was a perfect environment for entrepreneurship. Had feudalism existed in Venice as in Western Europe, merchants like Antonio would never have prospered as they did.

Since wealthier Venetians could not show off their titles or land, they had to find other areas of life to be ostentatious. Venetians used their immense wealth to build and decorate churches and piazzas, ostentatious palazzos and buy art to show off. This promoted a further growth in consumerism which expanded mercantile profitability through the sale of luxury goods. These were accessible to only the wealthiest in England but remained a fascination to the rest of the population.

Scene 4: Mercantile thievery and a darker secret

The merchants were not simply traders but thieves too. Probably best known are monuments stolen from the East: The Lion of Saint Mark, (from southern Turkey, dating back to 300 BC), the Horses of Saint Mark (stolen during the Sack of Constantinople at the end of the Fourth Crusade, supposedly previously displayed at the Roman Hippodrome) and the Sarcophagus of Saint Mark, stolen by Venetian merchants from Alexandria in 828 AD.



More importantly they 'stole' knowledge. They learned banking and accounting systems from the East along with their understanding of astronomy. The Venetian and other Italian merchants adapted to the Eastern ways of doing business. A good example is merchant Leonardo Pisan from Pisa (a rival Renaissance city state), known to us as Fibonacci. During his trips around the Mediterranean in the 13th Century, he discovered Hindu-Arabic numerals and incorporated their potential into European business transactions. Arithmetic as we know it today did not exist in Europe before the 15th Century yet had been explained by Persian astronomer Abu Ja'far Mohammed ibn Mûsâ al-Khowârizimî in 825 AD, in his book which translates to 'Rules of Restoration and Reduction'. Fibonacci's techniques were absorbed by Venice and helped merchants in increasingly complicated trading transactions. Early cheques, known as bills of exchange derived from the Arab term sakk. A knowledge of astronomy helped the merchants with navigation and enabled them to seek new markets. These new sophisticated banking skills were adopted by the Jewish community who provided financial support and loans to the Venetian merchants.

There was also a much darker side to Venetian trade that is often overlooked: the slave trade. The Renaissance slave trade directly led to the trans-Atlantic slave trade. The Venetian merchant Alvise Cadamosto described trading seven horses "which together had originally cost me about three hundred ducats" for one hundred slaves. At a relative rate of nine to fourteen slaves per horse, this was a very profitable deal. Cadamosto estimated that around 1,000 slaves were annually taken from the region of Arguim (modern day Mauritania) and sold in Lisbon and across Europe. By the mid-15th Century, Venice alone had a population of around 3,000 slaves (estimated population of Venice in 1500 was 1.5 million).

We do not know specifically what Antonio's merchandise was, but he traded across Europe and across the Atlantic (both England and Mexico are explicitly mentioned in the play). Judging by the awful levels of slave trade by Spanish merchants between Africa and Mexico, it is possible to believe that Antonio could have traded slaves needed by Spaniards in the New World to mine gold and natural resources, in exchange for goods that could only be found in the Americas like cocoa and maize, along with precious metals and gems discovered. In the play, we are led to believe that Antonio was a respectable character who we judge favourably, however, if he was involved in the slave trade, a 21st Century judgement of his character would be entirely different to that of a 16th Century interpretation.

Scene 5: State Protection

The state also protected Venetian survival; and the lives of the merchants. Since Venice is surrounded by water, one of its greatest weaknesses was the threat of plague. Although trade brought back lots of money and commodities to Venice, it also brought rats which carried diseases that Europeans had no immunity to. The Black Death wiped out almost half of the Venetian population when it arrived in January 1348. In response, the Venetian government introduced an institution of epidemic prevention forcing incoming ships to dock outside Venice for forty days. Cargo was washed with vinegar and placed in rooms of smoking herbs to purify the goods and kill parasites. This was a state paid institution, as forcing merchants to pay for this would have driven up prices, driving trade away from Venice. This system became known as quarantine, deriving from the Venetian words for forty days, 'quaranta giorni'. Plague prevention was essential to the survival of Venice. Plague would destroy populations, taking generations to recover, therefore, for Venice to survive, it needed its merchants, shipbuilders, glass blowers, tailors and builders to remain healthy.

In stark contrast, from 1592-1594, England had suffered a severe outbreak of plague. It had no plague control management or quarantine. Once again Venetian society was progressive compared to England.

Scene 6: L'Arsenale, shipbuilding and commerce

Antonio was a merchant. In his book 'Venice: A Maritime Republic', Frederic C. Lane examines in depth the "Commercial Revolution of Resident Merchants". A resident Venetian merchant lived and operated out of Venice rather than travelling on galleys with his shipments. To understand the character of Antonio, it is important to explain the misconceptions behind Venetian merchants. Notably, most did not own their own ships. The Venetian government crucially nationalised shipbuilding and then auctioned space on the ships to merchants for a voyage. More simply, several ships would travel between destinations and Venice at one time. The merchants had to bid to win storage space upon these galleys. This not only brought in more funds to the Venetian government but also limited the amount of control the merchants had over each other. It is also important to note that merchants did not necessarily work alone: but would club together for long periods of time to reduce the price of purchasing their merchandise abroad, these became known as "cartels". Joint ventures were very common. Many were between family members (and lasted the longest), some between friends; and some simply a professional contract (which often lasted about five years). Joint ventures allowed more money to be invested in a single voyage, meaning more and better-quality produce could be acquired abroad and then brought back to enlarge profits. Galleys sailed in convoys to their respective destinations, and shareholders of all the different galleys often came together and formed a single pool, called a maona. Therefore, when picturing Antonio, it is important to remember the limitations of his power as a merchant working alone.

One of the most important districts to merchants was L'Arsenale where shipbuilding occurred. In Alexander Armstrong and Dr Michael Scott's 'Italy's Invisible Cities', the pair investigate the productivity of L'Arsenale. Geographically, the district took up 10% of the entire city employing 16,000 workers. Dr Scott credited it as being the "first production line in Europe". Three ships were finished every day and at the turn of the 16th Century, during the reign of Henry VII in England, the Venetian navy was made up of 3,000 vessels; England had only five. Venetian shipbuilding was fundamental for the prosperity of merchants such as Antonio. The naval ships protected Venetian trading convoys but L'Arsenale also built the ships that merchants bid for cargo space on. The more ships that were produced, the better it was for Venetian merchants.

To exploit the demand for luxury goods, Venetian merchants had to supply more. The Venetian government (made up of wealthy merchants) changed the type of ships that L'Arsenale produced. Previously, narrower galleys, propelled by oars were mass produced. However, Venice turned to Cogs which were far larger, masted and round-bottomed. This increased the capacity Venetian merchants could transport to three hundred barrels (one barrel contained nine hundred litres). By the end of the 15th Century, Venice was producing the Caravel. Once again influenced by the Arab world, the Caravel could carry four hundred barrels, travel far faster than Cogs and it was strong enough to travel across oceans as well as calmer waters. This increased the range of trading destinations including the Americas as well as Africa, Europe; and the Near, Middle and Far East. Merchants like Antonio would have benefited immensely from the increased capacity of the trading vessels employed.

In Act 1 Scene 1, the ships that Antonio uses are named as Argosies and Andrews. Argosies were produced in Ragusa, modern-day Dubrovnik and



Andrews (the English translation) were ships flying the flag of Andrea Doria, a powerful Genoan. In Venice, in order to protect state income, it was illegal to hire or purchase ships that were built outside of the city, therefore, Antonio was committing a crime by using these ships. Ragusa and Genoa, alongside Spain and England were Venice's biggest rivals and due to the close access to oak, Ragusa was also a major ship building rival. This historical detail suggests that Antonio was not an honest merchant.

This commanding shipbuilding and naval prowess would have been of significant interest to a 16th Century English audience, particularly in light of the recent defeat of the Spanish Armada in 1588. The threat of another Spanish-led Catholic invasion via the sea remained constant for decades after.

Scene 7: The Rialto

One of the most famous lines from the 'Merchant of Venice' is "what news on the Rialto". The Rialto performed many roles in Renaissance Venice. It was the biggest market in Venice, but also the place to go for news and gossip. The Rialto area grew from its original size. As demand increased, land was cleared to the west of the original site to do this. The line from Act 3 Scene 1 highlights the importance of the Rialto for conversation. In Peter Ackroyd's 'Venice: Pure City', Ackroyd explains that at the base of the Rialto bridge "patricians met and mingled within a loggia or open-sided gallery". Lane stresses in his book that "a court sentence forbidding a merchant to go to the Rialto was the equivalent of putting him out of business". The importance of the Rialto to a merchant like Antonio would have been enormous. In addition to the communication between Venetians, letters to and from trading destinations were exchanged advising precisely how markets were performing abroad in order to maximise profits. In addition, the first city bankers set up stalls at the Rialto to allow the easy transaction of money between two parties, without the need for carrying cash in hand. The importance of The Rialto to Antonio and other merchants cannot be underestimated.



Scene 8: Writing for an English audience

Shakespeare had to write plays that appealed to his audience and patron. This was not easy. During the serious outbreak of plague in 1592, the authorities closed all playhouses in London to prevent the spread of disease, this was not lifted until 1594 and it continued to deter people from attending. By 1595, a new acting company had formed under the patronage of Queen Elizabeth's Lord Chamberlain, Henry Carey, First Baron Hudson and Shakespeare is named as a joint payee. He was now an actor, writer and shareholder in a prestigious theatre company, but he was required to fill The Theatre to pay costs. It is with little surprise that he would turn to Venice with its wealth of exotic goods, that were slowly reaching England as a setting for a play. For the Elizabethan audience, Venice was a symbol of mystery, luxury and intrique.

Venice represented wealth beyond belief to the English masses, palazzos built of marble and stone whilst most of London remained predominantly comprised of wooden structures. Venetian clothing was vibrant in colour and the materials of the finest quality, unlike English clothes which were dull by comparison; and patterned only by embroidery.

Elizabethan society had suffered repression and religious upheaval and was under the autocratic rule of a hereditary monarch, whilst Venice had a republican system that was merit based. By 1596 Queen Elizabeth was elderly; she had survived a series of assassination attempts and intrigue was rife regarding who would succeed her. Elizabeth censored any mention of succession in plays and in court, only further proving the autocratic nature of her reign.

Therefore, the setting of Venice would have been extremely appealing to English audiences and Shakespeare would have been able to draw in larger crowds. The subtleties of his Venetian setting



appealed to both Elizabethan court intellectuals and the masses at large.

Act 2: Antonio and Shylock (What does history tell us about the relationship between Antonio and Shylock?)

Scene 1: The Ghetto

The relationship which defines Shakespeare's 'The Merchant of Venice' is that of Antonio and Shylock the Jew, who is often portrayed as scheming and evil (although modern interpretations frequently depict Shylock as the victim of anti-Semitism). There are several reasons why Shakespeare may have chosen Shylock as his villain. These lie in the historical context in which Jews were viewed in Venice and England in the late 16th Century. There is no evidence to show that Venetian society was not anti-Semitic, however, by comparison to the rest of Europe, Venetian Jews were more integrated into society because they provided a vital service to the economy. The Jews had always been subject to persecution in Europe, however, this was exacerbated in 1555, when Pope Paul IV attacked the Jewish people in a Papal Bull:

"the Church only tolerates Jews in order that they may bear witness to true Christian faith"

The consequence of the Papal Bull was to entrench and endorse anti-Semitism in Europe. Jews were forbidden from owning property, confined to ghettos and required to wear a yellow badge. The play shows us that Shylock lived in a ghetto and was subject to prejudice but in the late 1500's Venetian Jews were treated differently to those in other parts of Europe. In Peter Ackroyd's 'Venice Pure City', he credits Venice as the first European city to benefit from "city-planning, with the deliberate 'zoning' of industries and activities along the peripheries of the city". A perfect example is the Jewish quarter of the city, or the ghetto which nowadays gives connotations of segregation and poverty, but it was in fact an important financial district in Venice; providing usury, financial services and pawnbroking. By the commercial restrictions placed upon them, which limited the professions they could undertake; the Jewish community became highly skilled in finance and accounting. The Ghetto was essential for the Venetian economy.



The Venetian ghetto was a designated area where Jews could live and practice their religion. The Ghetto being locked is deeply unjust, however, it was not an uncommon occurrence at the time. The same occurred to Venetians in Alexandria to prevent religious unrest during times of increased religious zeal and drunkenness. Lane said in 'Venice: A Maritime Republic': "this was explained, as was the locking in of Venetians in Alexandria at night and during hours of prayer on Muslim holy days by claiming that it was necessary to protect them from fanatics and to prevent scandal". Venice never recorded any incidents of major religious unrest in the city, suggesting that Christians and Jews lived in relative harmony together. As the Jewish population increased, the Venetian government granted more space and a burial ground in the lagoon to improve the quality of life for Venetian Jews.

Venetian Jews were rarely forced to wear yellow clothing, and many Jews moved to Venice as attitudes were more lenient towards them there than the rest of Europe. In Thomas Coryat's 'Description of Venice' from 'Coryat's Crudities, written in 1611 he describes his visit to a synagogue in the Jewish ghetto:

"In the room where they celebrate their divine service, no women sit, but have a loft or gallery proper to themselves only, as ever I saw, and so gorgeous in their apparel, jewels, chains of gold, and rings adorned with precious stones, that some of our English countesses do scarce exceed them, having marvellous long trains like princesses that are borne up by waiting women serving for the same purpose. An argument to prove that many of the Jews are very rich."

However, one reason for anti-Semitism amongst merchants like Antonio during the late 16th Century is overlooked. Due to violent persecution in Spain, many Jews were forced to convert to Christianity and fled to Portugal, one of these was a man christened as Joao Miquez, although he later took the Jewish name of Joseph Nasi. This group of Jews were known as 'New Christians' and many were wealthy merchants. Nasi was very influential due to friendships he had in trading ports across Europe, including a cousin in Venice. Nasi based his operations in Constantinople and used his connections with other Jews and New Christians across Europe to work together and increase profits. In Constantinople, this caught the ear of the Sultan. Nasi gained influence, was appointed Duke of Naxos (a title which before the following invasions was held by a Venetian family) and (it was rumoured) instigated the Ottoman invasion of Cyprus in 1571, which until this point had been part of the Venetian Empire. The actions of Joseph Nasi created a areat deal of friction between the Venetian Jewish population and the merchants who had to compete with Nasi. Had Shakespeare been aware of the tension it would make sense that 'The Merchant of Venice' featured heightened levels of anti-Semitism, from Antonio towards Shylock.

Scene 2: Usury

Antonio's hatred for Shylock was based on religious grounds, particularly Shylock's occupation as a usurer. In the play we are led to believe that Venetians hated Jews because they practiced usury which was a sin. However, this is a naïve view of the relationship between late 16th Century Venetians and the Jewish community living there. Christians were forbidden from practising usury by order of Pope Clement V in 1311, who declared usury as heresy. Dante in his 'The Divine Comedy' (written between 1308 and 1320) placed usurers in the 'inner ring of the Seventh Circle of Hell'. As Christians could not lend money for profit, Jews were encouraged to practice usury to support the Venetian mercantile economy. It was necessary for Venetian merchants to borrow money, and desperate Venetians like Shakespeare's Bassanio would have relied on borrowing from men like Shylock to support themselves in the consumerist Venetian economy. It was also the Jewish community that provided the expertise in banking. Economically, Venice was structured to benefit money making and a merchant would have needed a good relationship with a usurer. This is not reflected in the play.

Scene 3: Elizabethan anti-Semitism

It is important to assess attitudes towards Jews in England to understand the play better. Shakespeare needed an evil character in the play and who better to use than someone from a religious group that were banned in England and viewed as traitors. Jews were expelled from England in 1290 and not fully accepted till the 19th Century under Benjamin Disraeli. There was strict government censorship of the theatre in England regarding matters of religion and state. In 1581 it was decreed that all plays must be have a trial run in front of Master of the Revels, Edward Tilney, before public viewing occurred. It is likely that Shakespeare would have been arrested and potentially executed had he created a Jewish character in one of his plays that was not evil. Therefore, despite suggestions that Shakespeare held progressive views about equality, both gender and racial, ("I am a Jew" speech from Act 3 Scene 1) he would have had to refrain from trying to export his ideas through his plays.



In addition to abiding by censorship, Shakespeare had to write plays to fill theatres. He needed believable, credible villains. By the late 16th Century, Elizabeth I had survived several assassination attempts but in 1594 (only 2 years before the play was written) an attack involved a Jewish convert Dr Rodrigo Lopez, the Queen's physician. He was offered 50,000 ducats to poison the Queen as part of a Catholic plot supported by the Spanish court. The plot was discovered, and all perpetrators were beheaded. Against a background of such treachery, it would be easy to portray Jews in a negative light in England at the time.

Epilogue – My conclusions

My intention was to examine Shakespeare's The Merchant of Venice with specific reference to Venetian history to see if it could further my understanding of the play. Also, I sought to use historical evidence to better comprehend the relationship between Antonio and Shylock.

Analysis of all the characters will continue to develop, however, in my opinion, modern-day interpretations are not useful and are in fact a barrier which prevents people from understanding the Elizabethan perspective, and from considering why characters were depicted in a certain way.

In Elizabethan society, if you were not English, Protestant and white you were an outsider in a deeply prejudiced society. This is summed up best by Portia's description to Nerissa about her failed



suitors (Act 1 Scene 2). Portia derides all of her foreign suitors: The Duke of Saxony's nephew as an alcoholic, the Scottish Lord as dependent on help from the French, the Neapolitan Prince as Century would have understood his use of the trading ships named Argosies and Andrews and understood that this made him a less than honest merchant, indeed a tax dodger and potentially a



illegitimate, Monsieur le Bon the French noble as so obsessed with fencing that he would fence his own shadow; and in Act 2 scene 7 the Prince of Morocco "a gentle riddance. / Draw the curtains, go. / Let all of his complexion choose me so". She dismisses him based on his skin colour. Portia's description of her failed suitors and views of them are stereotypically racist. This was not only acceptable but humorous to an Elizabethan audience. It is therefore not surprising that Shakespeare uses anti-Semitism between Antonio and Shylock, which was more accepted then than now.

In understanding the play, it is important to appreciate the audience that Shakespeare was writing for. Theatre in England in the late 16th Century was in its infancy and deeply censored. It was vital that he wrote plays that would appeal to an audience that was poorly educated, had suffered extreme religious turmoil, was institutionally anti-Semitic and fearful of Catholic and foreign invasion. With this background Shakespeare used stereotyping and prejudiced behaviour in his characters to appeal to his audience. Venice represented a cosmopolitan, republican society with wealth, luxury and intrigue by comparison to post feudal England with an autocratic monarch. This stark contrast again captured the imagination of his audience.

Although Antonio is portrayed as the hero, he is by no means a truly honest or decent character. 21st Century audiences see him as a bully and anti-Semitic but an English audience in the late 16th slave trader. However, Antonio's anti-Semitism towards Shylock would have been socially acceptable in 16th Century England. The notion that he traded slaves was also acceptable and seen as no more than legal business. Without a historical interpretation, this is lost to a modern audience.

An audience with such low literacy rates, could not have understood a multi-dimensional character and all the subtle references towards a more equal society. It is more likely that this is simply the way the play is interpreted today; we hope to find origins of egalitarian beliefs in places where there may not be any.

The play was, in Elizabethan England, a comedy. Anti-Semitism, gender inequality, role reversal and racism were the backbone of the humour in this play. The characters are designed to be twodimensional, over simplified, in order to add to the humour based around stereotyping. The idea that all Jews are usurers and evil, Venetians are either merchants or wasters and that all women are fair maidens waiting to be rescued by a handsome prince, in its simplest form is how the play was designed to appeal to the uneducated masses and not to a modern-day critic.

Above all, Shakespeare needed to write plays that would fill theatres. The survival of his players and himself depended upon the success of his plays. He had to write in a way which would avoid censorship and be socially acceptable. Therefore, 'The Merchant of Venice' must fulfil certain criteria: particularly in the relationship between Antonio and Shylock. Jews were outsiders and were social pariahs in 16th Century England, Shylock could never have been anything other than the villain. The play itself may be an enigma of concepts and ideas: gender equality, role reversal, racial equality and the ability to add comedy in between. However, it is essential to remember that this is how we interpret it now, centuries after and based on modern-day social attitudes.

Above all else, Shakespeare's 'The Merchant of Venice' is by no means an accurate history of Venice, but it provides a useful insight into social attitudes towards Venice and perceived "outsiders" in English society towards the end of the 16th Century.

Application of the metallic ratios to stretching by paired concentric circles

Henry Thake

Abstract

This paper proves, by means of partial differentiation and graphical analysis, the two dimensional stretching caused by four discrete circular orbits of two points is optimally mapped by use of the metallic ratios of the general formula

 $a + \sqrt{a^2 + 4}$

or

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$$a + \frac{1}{a + \frac{1}{a$$

1 FOUNDATIONS

1.1 Introduction

The metallic ratios are a series of irrational numbers whose name, in part, is the result of their most famous and leading value, the golden ratio. The golden ratio, the most irrational of all numbers, is without doubt the most well known example of a metallic ratio, crediting its rise in significance with the work of Fibonacci and his sequence. Anyone who has ever used a nail file will also be practiced, if not necessarily familiar, with the silver ratio and its

*Diagrams produced by Maxim Webb, Graphs plotted online using Wolfram Alpha and Desmos

relationship with circles of shared points on their circumference. Whether written as a continued fraction or in surd form, both of these constants are underpinned by a eloquent and concise sequence. The golden ratio is perhaps best known as being the fraction $\frac{1+\sqrt{5}}{2}$, whilst the silver tends to be shown as $1+\sqrt{2}$ and, whilst not instantly obvious, this generalises as $\frac{a+\sqrt{a^2+4}}{2}$.¹ A similar pattern holds true of the continued fraction form, using the Pringsheim notation, $a + \frac{1}{|a|} + \frac{1}{|a|} + \frac{1}{|a|} + \dots^2$

As mentioned above, the most popularised use of the golden ratios is in the Fibonacci sequence, and at that, the spiral generated as a result.³ Unsurprisingly, as with the rest of the generality of this series, the spiral of the golden ration is not unique and can be generalised for the metallic ratios, each using its own unique rectangle.⁴ The stretching of a metallic ratio can be related to the scaling of sides of the rectangle as it progresses about its cycle. Then could it be that the optimal way to stretch a substance is based on a metallic ratio?

In spite of the seemingly abstract nature of this topic, the most interesting facet is the interdependence with everyday life, where the metallic ratios appear in all sorts of ways. Whether cutting nails into a shorter arc or attempting to compress square arrangements into triangular ones^{*}, the presence of these ratios should not be undervalued, nor more importantly, go unnoticed.

*Whilst technically the sequence of

square numbers that map to triangular numbers is not a member of the metallic ratios, it is by no means a difficult task to jump to the correct pattern for these values. If the list of numbers that when squared also make a triangular number are listed sequentially, the series: $1,6,35,204,1189,\ldots$ can be seen to be the integer approximation of $(3+2\sqrt{2})^{n-1}$ where n is the position in the series of the squaring number.⁵ This constant is generated by a similar formula to that of the metallic ratios, only it is in the form $\frac{b+\sqrt{b^2-4}}{2}$. This is, in essence, the anti-metallic ratio, with the infinite continued fraction in Pringsheim notation of $b - \frac{1}{|b} - \frac{1}{|b} - \frac{1}{|b} - \ldots$

1.2 Key Mathematical Results

It is taken, without proof, that both the chain rule $^{\rm 6}$

$$\frac{du}{dv} = \frac{du}{dx} \times \frac{dx}{dv}$$

and quotient rule

$$\left(\frac{u}{v}\right)' = \frac{v.u' - u.v}{v^2}$$

are true statements.⁷

Additionally, the gradient function of any three dimensional graph of the general form f(x,y) can be differentiated to a vector in the form⁸

$$\nabla f(x,y) = \begin{bmatrix} f_x(x,y) \\ f_y(x,y) \end{bmatrix}$$

If we let the function $\Phi(x) = \frac{x + \sqrt{x^2 + 4}}{2}$ it follows that

$$\frac{1}{\Phi(x)} = \frac{2}{x + \sqrt{x^2 + 4}}$$
$$\therefore \frac{1}{\Phi(x)} = \frac{2(x - \sqrt{x^2 + 4})}{x^2 - (x^2 + 4)}$$
$$\therefore \frac{1}{\Phi(x)} = \frac{-x + \sqrt{x^2 + 4}}{2}$$

$$\therefore \frac{1}{\Phi(x)} = \Phi(-x)$$

Perhaps the most intriguing results are obtained from second order partial differentiation, namely that if we use the same the same general functional form of f(x,y), then we typically see that if $f_{xy}(x,y)$ and $f_{yx}(x,y)$ are continuous at (a,b), then $f_{xy}(a,b) = f_{yx}(a,b)$, i.e. that the order of partial differentiation makes no impact of the result of a second order partial differentiation. This property is no specific to functions of this type, as any such function will have the property that under *a* orders of partial differentiation, any combination of b differentials with respect to x and a - b with respect to y will have the same value. The result also holds true of k different variables within the function. To show the outcome of second order partial differentiation concisely, we shall produce it as such

$$\nabla^{II} f(x,y) = \begin{bmatrix} f_{xx}(x,y) \\ f_{xy}(x,y) \\ f_{yy}(x,y) \end{bmatrix}$$

2 <u>MODEL</u>

The prime example of stretching by concentric circles is found in the field of confectionary, more specifically, taffy-pulling. The process of taffy making starts by the caramelisation of sugar in solution, which is a gentle process where movement should be kept minimal to prevent crystallisation. However, the consistence of taffy is much softer and less brittle than that of caramel or toffee, which occurs through aeration. For decades, this process was done by hand until the mechanical revolution of the confectionary industry reduced the workload of confectioners. The system of pulling taffy operates on a system of fixed and orbiting pins that drag and stretch the taffy to trap air within folds and soften the texture.⁹ The best way to model this is a set of concentric circles.



Let there be two points in the complex plane P and Q such that

$$P = x + iy$$

and

$$Q = w + iy$$

$$w, x, y \in \mathbb{R}$$

Let us construct two loci about each point: A, B, C and D such that:

$$\begin{cases} A \\ B \\ C \\ D \end{cases} = \begin{cases} |z - P| = R \\ |z - Q| = R \\ |z' - P| = r \\ |z' - Q| = r \end{cases}$$

$$R > r; R, r \in^+ \mathbb{R}$$



Figure 1: *Complex Loci constructed as described, with points labelled*

Let each locus have two points located collinear to the line connecting PQ at π^c apart. Let the point with the larger real part be denoted by the point X_2 and the other by X_1 .

 $\therefore A_1B_2$ is the greatest stretch of the material, whilst for the converse it is C_1D_2 . Let the ratio $A_1B_2 : C_1D_2$ be some variable $\Phi : 1$, $\Phi \in \mathbb{R}$.

Let the distance between the points P and Q be a length d, $d \in \mathbb{R}$.

2.1 Manipulation

In order to achieve an optimum solution to the problem, we must define a set of variables that we need to maximise and minimise respectively. Once a system of equations are attained, it should be possible to minimise the number of total variables to gain a single equation for each defined event.

Based on the assertions made in the previous section

$$\therefore A_1 B_2 = \Phi(C_1 D_2)$$

$$\Rightarrow 2R + d = \Phi(2r + d)$$

$$\therefore 2(R - r\Phi) = d(\Phi - 1)$$

$$\therefore d = 2\frac{R - r\Phi}{\Phi - 1}$$

 $\Phi > 1, R \ge r\Phi$

Now, let us consider a general length for R. From the diagram, it is shown that

$$R = d + kr$$

$$k \in \mathbb{R}; -1 \le k \le 1$$

$$\therefore \Phi(2r+d) = 2R + d$$

$$\Rightarrow 3d + 2kr = \Phi(2r+d)$$

$$\therefore d = 2r\frac{\Phi - k}{3 - \Phi}$$

 $\Phi < 3; \Phi \ge k$

This enables us to generate the following set of simultaneous equations

$$d = \frac{2(R - r\Phi)}{\Phi - 1} \tag{1}$$

 $\Phi > 1; R \ge r\Phi$

$$d = \frac{2r(\Phi - k)}{3 - \Phi} \tag{2}$$

$$\Phi < 3; \Phi \ge k$$

By subtracting (2) from (1), we achieve the equation

$$\frac{R(3-\Phi) - r([2-k]\Phi + k)}{(\Phi-1)(3-\Phi)} = 0$$

$$\therefore R = r\frac{(2-k)\Phi + k}{3-\Phi}$$
(3)

$$1 < \Phi < 3$$

or similarly, bound by the same parameters of Φ

$$R = r \frac{2\Phi + (1 - \Phi)\lambda}{3 - \Phi}$$

Moreover, as $R \ge r\Phi$

$$\Rightarrow \frac{(2-k)\Phi+k}{3-\Phi} \ge \Phi$$
$$\therefore \Phi^2 - (k+1)\Phi + k \ge 0$$
$$\therefore (\Phi-k)(\Phi-1) \ge 0$$
$$\therefore \Phi \le k, \Phi \ge 1$$

From this set of solutions to the simultaneous equations, we can attempt to solve by several means: algebraic approximation, partial differential equations and graphical analysis.

2.2 Approximation

In this limiting system, to ensure that the full conditions of the model are met, when the value of d = 0, the four separate circles must totally overlap to form one circle instead of four, where R = r and $k = \Phi =$ 1. As the value of d is steadily increased, the circles must instantaneously become discrete, so the value of R is relatively increased to a fixed value of r. The result of this variation in values of R and d is that the value of Φ can be varied with a changing value of k, which can be kept independent of all other values. Thus, there can be no other way of other way of writing k and Φ as anything other than the variables in which they represent, as their values influence the other variables, and the other variables cannot influence their value, except in the limiting scenario. The limiting case is impossible as it contradicts the model, so the only time when k and Φ are fixed is invalid. This result also eliminates the possible result where $\Phi \leq k$, as for the overall model to hold, the value of $\Phi > 1$ except in the limiting case where $\Phi = 1$.

Let us establish a basis for further calculation by approximating algebraically where

the solution may exist. Let us assume that a series of good, possibly optimum, solutions occur for Φ that obey the general formula

$$\frac{a+\sqrt{a^2+4}}{2}$$

 $a \in \mathbb{R}$

Now, given known equations for d and R, we can see that

$$d = 2r \frac{\frac{a + \sqrt{a^2 + 4}}{2} - k}{3 - \frac{a + \sqrt{a^2 + 4}}{2}}$$
$$d = 2r \frac{(a - 2k) + \sqrt{a^2 + 4}}{(6 - a) - \sqrt{a^2 + 4}}$$

and also

$$R = r \frac{(2-k)\frac{a+\sqrt{a^2+4}}{2} + k}{3 - \frac{a+\sqrt{a^2+4}}{2}}$$

$$R = r \frac{(2k+2a-ak) + (2-k)\sqrt{a^2+4}}{(6-a) - \sqrt{a^2+4}}$$

which can be rationalised to

$$d = r \frac{(3a + ak - 6k + 2) + (3 - k)\sqrt{a^2 + 4}}{8 - 3a}$$

and

$$R = r \frac{(3a - 2ak + 2k + 2) + (3 - k)\sqrt{a^2 + 4}}{8 - 3a}$$
$$\therefore R - d = r \frac{k(8 - 3a)}{8 - 3a}$$
$$\therefore R = d + kr$$

This is a pleasing result as it does mean, at the very least, that when a metallic ratio is used as the stretching factor, it doesn't create a contradiction or break any fundamental theorems. For the present, we will thus assume that any value in this range is indeed a good approximation of the optimum solution.



Regardless of Φ , the whole system of variables can be reduced to the simple set based on three variables, including Φ itself.

$$\begin{cases} r \\ k \\ \Phi \\ R \\ d \end{cases} = \begin{cases} r \\ k \\ \Phi \\ r \frac{(2-k)\Phi+k}{3-\Phi} \\ 2r\Phi \frac{1-k}{3-\Phi} \end{cases}$$

We can now take into account the values of this series of functions when indeed we take $\Phi = \frac{a+\sqrt{a^2+4}}{2}$

$$\begin{cases} r \\ k \\ \Phi \\ R \\ d \end{cases} = \begin{cases} r \\ k \\ \frac{a + \sqrt{a^2 + 4}}{2} \\ r \frac{(3a - 2ak + 2k + 2) + (3 - k)\sqrt{a^2 + 4}}{8 - 3a} \\ r \frac{(3a + ak - 6k + 2) + (3 - k)\sqrt{a^2 + 4}}{8 - 3a} \end{cases}$$

3 ANALYSIS

3.1 Partial Differentiation

We now have a known multivariable equation for R in terms of r, k and Φ which can be written both in the forms of

$$R = r \left[\frac{(2-k)\Phi + k}{3-\Phi} \right]$$

or alternatively

$$R = r \left[\frac{2\Phi}{3-\Phi} + \frac{1-\Phi}{3-\Phi} k \right]$$

However, as r is the only variable that is totally independent of the other variables, it can be treated as a constant, and hence, R is now only calculated by a two variable equation. However, the other two variables cannot be eliminated from the equation, so it cannot be singularly differentiated, so instead, we must use partial differentiation.

$$\nabla R(k, \Phi) = \begin{bmatrix} \frac{\partial R}{\partial k} \\ \frac{\partial R}{\partial \Phi} \end{bmatrix}$$

By twice using the quotient rule, we can easily find the two differentials of ${\sf R}$

$$\frac{\partial R}{\partial k} = \frac{1 - \Phi}{3 - \Phi}$$

and also

$$\frac{\partial R}{\partial \Phi} = \frac{(3-\Phi)(2-k) + \left[(2-k)\Phi + k\right]}{(3-\Phi)^2}$$
$$\Rightarrow \frac{\partial R}{\partial \Phi} = \frac{6-2k}{(3-\Phi)^2}$$

Overall, this enables us to generate a vector for the gradient of the surface R of

$$\nabla R(k,\Phi) = \begin{bmatrix} \frac{1-\Phi}{3-\Phi} \\ \frac{6-2k}{(3-\Phi)^2} \end{bmatrix}$$
(4)

In order to be able to contrast the gradient of any two points on the surface R, we need to convert the two vectors that form the single vector $\nabla R(k, \Phi)$ to a single vector in the form of a complex number $\alpha x + \beta y$. In order to achieve this we have to calculate the cross-product of the two component vectors of $\nabla R(k, \Phi)$. However, this is not a necessary step, as based on the known limits of k and Φ , we can see that the $\frac{\partial R}{\partial k} < 0$ and $\frac{\partial R}{\partial \Phi} > 0$. Additionally, as both differentials are continuous functions within the given ranges of k and Φ , and are a factor of -1 in separation, there is no point on the surface R that has zero curvature, so no optimum solution for R in terms of the gradient of the surface $\nabla R(k, \Phi)$.

Let us now consider whether an optimum solution exists for the other multivariable function in r, k and Φ , that is d. We have a known function for d in the form

$$d = 2r\Phi \frac{1-k}{3-\Phi}$$

Let us again rewrite this equation to make the partial derivatives more intuitive, that is as follows

$$d = \frac{2r\Phi}{3-\Phi} - \frac{2r\Phi}{3-\Phi}k$$

and

$$d = 2r \frac{(1-k)\Phi}{3-\Phi}$$

From this we can say the following about the partial derivatives of d with respect to k and Φ if we again treat r as a generic constant as we did with the partial derivatives of R.

$$\frac{\partial d}{\partial k} = -\frac{2\Phi}{3-\Phi} \tag{5}$$

and

$$\frac{\partial d}{\partial \Phi} = 2 \frac{(1-k)(3-\Phi) + (1-k)\Phi}{(3-\Phi)^2}$$
$$\therefore \frac{\partial d}{\partial \Phi} = 6 \frac{1-k}{(3-\Phi)^2} \tag{6}$$

If we treat the gradient as the following vector

$$\nabla d(k, \Phi) = \begin{bmatrix} \frac{\partial d}{\partial k} \\ \frac{\partial d}{\partial \Phi} \end{bmatrix}$$

This leads us to make the following conclusion about the gradient of the function for d

$$\nabla d(k, \Phi) = \begin{bmatrix} -\frac{2\Phi}{3-\Phi} \\ 6\frac{1-k}{(3-\Phi)^2} \end{bmatrix}$$
(7)

Based on the known limits of k and Φ that were established in section 3, we can establish some properties of ∇d without the requirement to take cross product if the two component vectors. As $0 < \Phi < 3$, we can see that the value of the $\frac{\partial d}{\partial k} < 0$. For the second vector, we can see that the denominator must always be greater than zero as it is a square number whose value cannot be zero. It does appear, at a glance, that this function may actually have a turning point in the partial derivative from Φ , as when k = 1, the numerator, thus the whole fraction equals zero. However, as proved at the start of section 5, this result is the limiting case, so does not apply to the model. Thus, based on the known limits that -1 < k < 1, we know that the numerator must also be greater than zero. We thus achieve the same general result for ∇d of when we took the partial derivatives of R, where no true optimum solution exists.

Out of interest, let us consider the second order partial derivatives of the function $R(k, \Phi)$ to see how the surface is changing with respect to its variables. We can, as an aside, quickly demonstrate the interesting property of second order partial derivates being equal irrespective of order here. Let us first consider the partial derivatives of $R_k(k, \Phi)$.

$$R_{kk}(k,\Phi) = \frac{\partial}{\partial k} \frac{1-\Phi}{3-\Phi}$$
$$R_{kk}(k,\Phi) = 0$$

and also

$$R_{k\Phi}(k,\Phi) = \frac{\partial}{\partial\Phi} \frac{1-\Phi}{3-\Phi}$$
$$R_{k\Phi}(k,\Phi) = -\frac{2}{(3-\Phi)^2}$$

Now let us consider the partial derivatives of $R_{\Phi}(k, \Phi)$.

$$R_{\Phi\Phi}(k,\Phi) = \frac{\partial}{\partial\Phi} \frac{6-2k}{(3-\Phi)^2}$$
$$R_{\Phi\Phi}(k,\Phi) = 2\frac{6-2k}{(3-\Phi)^3}$$

and also

$$R_{\Phi k}(k,\Phi) = \frac{\partial}{\partial k} \frac{6-2k}{(3-\Phi)^2}$$
$$R_{\Phi k}(k,\Phi) = \frac{2}{(3-\Phi)^2}$$

Thus, it has been shown that within this example, that as expected $f_{xx}(x,y) = f_{yy}(x,y)$.

Anyhow, let us return to the question of changing surfaces. If we condense the results of the second order partial differentiation, then we get the result below.

$$\nabla^{II} R(k, \Phi) = \begin{bmatrix} 0 \\ -\frac{2}{(3-\Phi)^2} \\ 2\frac{6-2k}{(3-\Phi)^3} \end{bmatrix}$$

This result does enable us to pick up one crucial result. Let us just consider from the function $R(k, \Phi)$ only performing partial differentiation with respect to the variable Φ . It is simple to prove that the n-th partial

derivative with respect to Φ has the general form $n! \frac{6-2k}{(3-\Phi)^{n+1}}$ which infers that for the values in the domain of the function, this term will be positive. Additionally, any n-th order partial derivative that has only a single differentiation by the variable k will certainly be negative and of the general form $-(n-1)!\frac{2}{(3-\Phi)^n}$. Any other combination of partial differentiations from this will always result in a value of zero. Thus it is proven that the function $R(k, \Phi)$ cannot be optimised for any value in its domain.

3.2 Graphical Analysis

Let us return to the function for R defined as the result in section 5, $R = \frac{(2-k)\Phi+k}{3-\Phi}$.



Figure 2: Simple contour of function, with lighting shading at greater R values. Contour produced on Wolfram Alpha.

In order to easily analyse this function, let us produce a contour graph to examine the increase in the value of R with the variation of k and Φ . From *figure 2*, and perhaps from an even fleeting inspection of the equation defining R, we can see that as the value of k increases, the value of R decreases when $\Phi > 1$ and increases when $\Phi < 1$. Moreover, also apparent from the equation, the



Figure 3: Detailed contour of function, with equivalent values of R marked, for values of R (left to right) : 0, 0.25, 0.5, 0.75, 1*, 1.25, 1.5, 2, 3, 4, 5, 10, 15, 20, 30, 50 and 100. *note, the contour of R = 1 is the line k = 3 so does not appear in this range. Contour produced on Desmos.

value of R will increase greatly as the value of Φ increases.

Now, let us examine the contour graph produced from figure 3 more closely. We can see that as the value of Φ increases, the corresponding value of R increases. However, this sort of relationship is not as apparent when analysing with respect to k. In fact, the effect of changing k on R is totally determinate on the value of Φ . However, as the minimum value of $\Phi > 1$, this makes the problem slightly easier, as the breaking point is at $\Phi = 0$, so the effect of increasing k is actually a decrease in R. Therefore, the value of k should be proportional to that of Φ , as the smaller value of R can be made from $\Phi = 2, k = 1$ than $\Phi = 2, k = -1$. The optimum solution to the problem would occur at the point where this beneficial pattern starts to be removed, and the effect of increasing k starts to lessen. However, this point will occur in a fairly broad area in the top right region of the *figure 3*. As shown in figure 3, in this designated area, all of the contour lines begin to cluster together as the function tends to infinity at $\Phi = 3$. This leaves two regions of near vertical contours of the function, one at $\Phi = 1$ and the other at $\Phi = 3$. This infers that the better solutions will exist in the region of $\Phi = 2$. In order to prevent any value of R from getting too large, it could be suggested that any solution in the region of $1.5 < \Phi < 2.5$ and where k is as close to its upper bound of one as possible, as this will always minimise R with respect to Φ , with a low value of Φ giving a less efficient but smaller machine, whilst a larger value of Φ and k giving a more efficient but grander contraption.

4 <u>CONCLUSIONS</u>

4.1 Discussion

The implications of the results are that no single systematic stretch is especially efficient when contrasted to its counterparts, and that with a fixed set of parameters, the relationship between metallic ratio and absolute system size will never have a region where the second derivative is less than or equal to zero. This poses the question of generality, something which has influenced much of the paper, by which if the number of fixed pivot points increases, does that offer an optimum solution, say with four fixed pins arranged in square plane. More pressing still, if it were possible to create a system of spherically rotating points in three dimensions, could a similar construct occur here, and would that have an optimum stretch series or absolute value? Again when expanding to the third dimension or higher, is it that equidistant points generate a higher optimum stretch per volume of space occupied, or is it more beneficial to have the same number of pins but using a lower dimension to perform the stretching, i.e. is a square plane better than a tetrahedral space?

Finally, there is the question of practicality. The limits for much of the paper, and indeed those set about by the results of the graphical analysis, were heavily influenced by the concept of practicality and real-world limitations, as infinitely large concentric circles, while theoretically not problematic, are hard to construct, let alone have an application in the world of confectionary. Is it, therefore, acceptable for the series of solutions generated to be viewed as being near optimal when the first value that could be used is, according to Section 1, "the most irrational of all numbers"? Moreover, can any number which can be expressed as an infinite continued fraction of constant denominator, i.e. $a + \frac{1}{|a|} + \frac{1}{|a|} + \frac{1}{|a|} + \dots$, and thus never closely approximate to a rational number, be deemed an acceptable real-world solution?

4.2 Conclusion

Based on the result of the partial differentiation of the function $f(k, \Phi)$, there is no true optimum solution to the problem, thus the best set of solution can be approximated through both the graphical and algebraic approximations of Φ . According to the algebraic approximation, a series of general solutions can be found for which all the known variables can be constructed from a variable and a scale factor when Φ is in the general form $\Phi = \frac{a+\sqrt{a^2+a}}{2}$. The graphical analysis lead to no better a result than that of the algebraic approximation, giving a general area of good solutions, rather than a single value. However, when both approximations are taken into account, both integer fed metallic ratios gave results that would suffice both solutions, and, as the function for the metallic ratios is continuous, every non-integer metallic ratio between would also give an adequate solution as confirmed by the graphical analysis, the algebraic calculations of the other variables could be used to deduce the vales of d and k.

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To what extent is the destabilization of German politics due to a populist surge resulting from the European migrant crisis?

Kit Edgecliffe-Johnson

From 1990 to 2017, Germany appeared as one of the most political stable countries in Europe. having never had a far right party even enter into its legislature. Since 1949 the German Parliament, the Bundestag, has been dominated by the Christian Democratic Union/Christian Social Union alliance (CDU and CSU respectively) and the Social Democratic Party of Germany (SPD). Since 1949 there hasn't been a year when one of the two was not in power. Germany has been held up as an example of moderate liberal democracy, being called a "beacon of stability" as recently as 2017. Indeed, in recent decades the German political system has seemed incredibly robust, electing the same Chancellor, Angela Merkel, every election period since 2005. Her steadfast hold on power as she promotes a pro-EU, immigration and Eurozone agenda has led her to be ranked as the most powerful woman in the world. This same political stability has enabled the German economy to flourish, becoming the largest economy in Europe, and fourth largest in the world.

Yet, in the aftermath of the 2017 German Federal Election, this entire German political structure came into question. The Alternative for Germany (AfD), a party founded only four years before the election, took 12.6% of the vote on an anti-immigration, anti-Islamic agenda, winning 94 of the 598 available seats in Bundestag. The two largest parties, the SPD and CSU/CDU, were forced into a second consecutive 'Grand Coalition', only the fourth in the history of the Federal Republic of Germany. This same coalition very nearly came apart this July, when emergency talks had to be held between Merkel and the CSU, whose leader, the German Interior Minister Horst Seehofer, threatened to resign unless stricter border controls were put in place. Under this pressure from the CSU, as well as factions within her own party, Chancellor Merkel was forced to bow to Seehofer's demands, walking back her headline policy of open borders in an embarrassing U-turn. Now labelled a lame duck leader, Merkel could not look less like the "Survivor Queen of Europe", that she was touted as but a few years ago. Numerous articles have referenced this as being due to the rise of 'populism', or 'populist parties' in Germany, pointing to the events of the 2015 migrant crisis as the catalyst for this surge - yet other factors such as a loss of confidence in the government, the European Debt Crisis, and the actions of the CSU itself, all have a role to play in this destabilization. Therefore, to what extent is the destabilization of German politics due to a populist surge resulting from the European Migrant Crisis?

At a basic level, we can see that the reason there has been a destabilisation of German politics is that fewer people are voting for moderate parties, and more people are voting for extreme parties. In 2013 the Free Democratic Party (FDP) only managed 4.8% of the vote, below the 5% needed to enter the Bundestag. This directly led to the first 'Grand Coalition' of the CDU/CSU and SPD, as the previous CDU/CSU and FDP coalition was no longer possible. The 2017 election saw the CDU/CSU, a centre-right alliance, lose 980,000 voters to the AfD, a far-right party, whilst the SPD, a centre-left party, lost 470,000 voters to the AfD. So, we can see that over the last decade, mainstream German parties have been weakened as their share of the vote has dropped, leading to coalitions between the two parties that traditionally oppose each other, the SPD and CDU/ CSU. This suggests that by understanding why voters have turned away from mainstream parties, we can understand why German politics has become less stable.

Some argue that one of the largest factors leading to voters turning away from mainstream German parties was the events of the Eurozone debt crisis. Beginning in 2009 as a result of Eurozone member states being unable to repay or refinance their government debt, the debt crisis had a huge effect on the German public consciousness. This was in part due to Germany being the largest economy in the Eurozone; it literally had the 'most to lose', but also was due to the large part Germany played in influencing the fiscal policy of the Eurozone. As noted in an article by the Independent, without German support none of the three bailout packages that the Greek economy received throughout the crisis would have been approved. Furthermore, Germany applied significant pressure to the European Central Bank (ECB) in terms of the policies it utilised throughout the crisis; it delayed the use of quantitative easing by six years, insisted on tighter fiscal rules, and blocked the ECB from



operating as a sovereign lender of last resort. With this in mind, it is undeniable that German policy had a large effect on the events of the Eurozone crisis.

This heavily interventionist stance taken by the German avernment under Chancellor Merkel and Finance Minister Wolfgang Schäuble was extremely unpopular at home. There was a worry that the actions of the government were sacrificing German interests in order to 'prop up' the European experiment, particularly in reference to the Greek economy. A 2011 YouGov poll showed that only 15% of Germans approved of the government performance during the Eurozone crisis, whilst 75% disapproved of the government's performance. 59% of the country opposed any further bailouts (two more would be enacted), and 58% supported a forced 'Grexit' - expelling Greece from the European Union. Needless to say, when the government refused to do so, approving further aid for Greece, its approval ratings suffered, falling 4% in August 2011.

The argument therefore follows, that the reason that Merkel's CDU/CSU lost votes in the 2017 federal election was due to dissatisfaction with the way that the government had approached economic crises such as the Eurozone debt crisis. Further, this dissatisfaction led to a loss of trust in the government, German citizens began to question whether or not Merkel's government was putting their interests first in the aftermath of the bailout.

Indeed, at first glance the data seems to support the claim that Germans had grown dissatisfied with their government; in March 2015, six years into the Eurozone crisis, 6/10 Germans still thought that Greece should leave the Eurozone. It is clear that the attitudes of the German people towards the Eurozone had not changed. Yet, under closer examination, we can see that despite this disconnect between the economic policies of the German government (who approved another Greek bailout package in 2015), and the feelings of German citizens, the CDU/CSU weren't actually punished at all in elections. In fact, in the 2013 federal election, in the middle of the eurozone debt crisis, the CDU/CSU won their best result since 1990, with nearly 42% of the vote, and only five seats short of an overall majority in the Bundestag.

We can further examine the 2017 federal election to show how there is little relationship between a government's economic success and their share of the vote. Data from the OECD showed that the German economy in 2017 grew at its highest rate in seven years, yet workers reported feeling "less confident" in the state of the economy, and politics in general. And, as stated before, when the 2017 election results emerged, it showed that despite this strong economic performance, the CDU/CSU and SPD had huge swings against them: -8.6% and -5.2% respectively.

This suggests that, despite popular disapproval of the government's economic policy in relation to the eurozone, the factors that largely contributed to a destabilization of German politics were not yet present. Or alternatively, that the factors were present, but needed a catalyst before they could have a large-scale effect on German politics. Therefore, we can see that the events of the Eurozone debt crisis, and the German government's response to the crisis, had little immediate effect on the destabilisation of German politics. However, it is possible that the crisis led to a loss of faith in Merkel, leaving open the risk of future, and further, dissatisfaction with her government.

As a result, we must instead examine other factors to fully understand the current situation in German politics. Given that the 2013 election showed high levels of support for the CDU/CSU and SPD coalition, yet both suffered heavily in the 2017 election, it is likely that a factor occurring between these two elections can explain just why German politics has become destabilised.

In this 2013-2017 time span, there is a single event that caused significant contention within Germany and Europe as a whole - the 2015 migrant crisis. These migrants were made up of forcibly displaced people (including 5.3 million Palestinians), economic migrants, and large numbers of refugees fleeing war, particularly from Syria and Afghanistan. From 2014-2015 the number of first time asylum applicants to the EU more than doubled, increasing from 626,000 to 1,322,000. Germany took in over 1.1 million asylum seekers in 2015 alone under Merkel's 'open-door' refugee policy. For Germany this was a huge increase on previous numbers - in 2016 Germany was taking in almost thirty times as many migrants as eight years before.

This 'open door' policy proved hugely unpopular in Germany for two reasons. The first was a worry that German culture was being washed away by this influx of refugees; the manifesto of the AfD demanded protection for "well-established cultural and regional traditions", whilst worrying that "mass immigration will "hasten the ethniccultural changes in society". This worry was largely concerned with the religious differences between the 60% Christian Germany, and the predominantly Muslim migrants entering into Germany. From 2010 to 2016 the number of Muslims in Germany increased by 6.3 million, from 3.8% to 4.9% of the population, a figure that the Pew Research Centre estimates to rise to 11.2% in 2050 with 'medium migration'. The popular response to this was mixed, and probably best explained by the "anxious middle" who make up 58% of Germany according to a study by Purpose in 2016. This group is neither overly positive nor negative about immigration, but generally sceptical about it as a concept.

The second reason why the German response to the miarant crisis proved unpopular was the perceived risk of terror attacks due to increased migration. The Global Terrorism Database shows an increase in fatalities resulting from terror attacks in Europe from 2014 onwards - by 2016 there were over twelve times as many fatalities in attacks as just two years before. Whilst many would argue correctly that the actual number of attacks have decreased in this period, the fact remains that there has been a spike in media reporting of terrorism - especially Islamic terrorism, fuelling fears across Europe. Regardless of whether or not terror attacks have actually got worse, and indeed, regardless of whether or not the migrant crisis is responsible, the fact remains that the three most important issues for AfD in 2017 voters were the fight against terrorism, fight against crime and the influx of refugees.

The above factors are best explained as having destabilised Germany through the splits they have produced within the existing political system. A meeting on the 1st of July 2018 saw Merkel contend with the CSU over just this issue, eventually having to walk back her 'open doors' policy in order to hold her coalition together. As a result, she agreed to implement stricter border controls that would prevent asylum seekers who are the responsibility of other EU members to be detained and sent back to the country responsible via 'transit centres'. This suggests that Merkel's handling of the migrant crisis itself is one reason that German politics has faced destabilisation.

This stricter immigration policy was championed by the CSU, who only field candidates in Bavaria, a region to the East of Germany. The party was under voter pressure to implement new restrictions on migrants, with 44% of CSU voters thinking Merkel hasn't taken popular concerns about migration sufficiently into account in her policies, whist immigration and integration as a whole has been the most important issue to voters since 2014. Indeed, when the CSU did propose stricter border controls it found large amounts of support from across the political arena, with the majority of both the CDU and the more left-wing SPD supporting this.

Yet, despite the cross-party support for more restrictive immigration policies, the media has been

quick to label such actions as 'populist', a term with decidedly negative connotations in today's political landscape. One such article from 'Social Europe' refers to the recent actions of the CSU as 'Germany's Populist Temptation', claiming that the statements of the CSU's leader, Horst Seehofer, are "perfectly populist", because he said "Islam does not belong in Germany." This would suggest that the political centre, particularly the CSU, is now populist, or at the very least, is enacting populist policies, and that by extension, populism is destabilising German politics.

In order to recognise either a party or policy as populist, a definition must be agreed upon which can be assessed against. Yet, this has historically proved difficult to do. To be sure, attempts have been made to provide a single definition, the most notable of which was a 1967 conference at the London School of Economics and Political Science (LSE), entitled "To define populism". Spaced over a two day period, political scientists from universities across Europe and the United States spoke and debated for hours on end, yet ultimately failed to come up with a unifying definition. However, between them they did manage to agree on a set of characteristics, which when tested against the policies and supporters of the CSU, will enable us to judge whether or not this faction within Germany is truly populist. These political characteristics, as identified by S.L. Andreski, Hugh Seton-Watson and Peter Worsley, are as follows:

1. A redistribution of wealth and/or power

2. Movement/protest on the behalf of the lower classes

2.1 Rural classes in particular

3. Preservation and idealisation of a traditional way of life

4. Organised by the intelligentsia and other elements on the behalf of the masses

5. Based out of a reaction to an externality (Usually due to outside elements imposing change

on a society)

An alternative, and far shorter definition, is the one put forward by Cas Mudde in "The populist Zeitgeist", who argued that there is no single ideology of populism, it is by its own nature a "thin ideology", simply an antagonism between the "pure people" and the "corrupt elite". So, in trying to understand whether or not a populist surge manifested through the CSU has destabilised German politics, we must judge the CSU against the above criteria.

To be sure, some of the characteristics identified at the LSE conference are present in the CSU. There is a significant part of Bavaria and Eastern Germany as a whole with a rural basis - Bavaria has the most



rural districts of any German states, whilst Eastern Germany has double the agricultural share of GDP of Western Germany. Furthermore, it could be argued that the proposed restrictions of refugees taken in to 200,000 per year are a preservation of a traditional way of life, through seeking to keep Germany culturally and ethnically homogeneous. Finally, this development in CSU policy against immigration was formed out of an externality - the 2015 migrant crisis.

However, when assessed against the 'thin ideology' of populism, that of an antagonism between the "pure people" and the "corrupt elite", we see that the CSU no longer looks nearly as populist. Fundamentally, no matter how the CSU might try to present itself, it has still been in power for the last 13 years of German government, and will likely continue for the immediate future, making it rather difficult for the CSU to try to claim its opposition to the political elite - given that it is the political elite. In fact, the CSU makes no attempt to do so, making no reference in its joint manifesto with the CDU to 'the political elite'. So, therefore we can see that the CSU is not a populist party, though its policies in reference to immigration might appear at first glance to be populist. This suggests that any idea of a 'populist surge resulting from the European Migrant Crisis' is mistaken.

exhibiting Instead. the CSU is nationalist characteristics, defined by Lowell W. Barrington as a combination of "the political idea of territorial self- determination, the cultural idea of the nation as one's primary identity, and a moral idea of justification of action to protect the rights of the nation against the other". Essentially, the CSU is attempting to appeal to a common sense of German and Bavarian identity to combat its falling vote share - through policies such as making it compulsory to have a Christian cross above all government buildings - designed to unify the Christian majority of Germany behind the CSU.

Yet, there remains a faction within Germany that does fit the previously set out characteristics of populism - the right-wing Alternative for Germany (AfD). The first page of its manifesto strikes out at the "political cartel" who are "secretly in charge" and "responsible for the misguided development of past decades", clearly calling for a redistribution of power from the political classes to the German citizens. The manifesto goes on to demand protection of "well-established cultural and regional traditions", with a "commitment to the traditional family as a guiding principle". It further notes that mass immigration will "hasten the ethnic-cultural changes in society", showing the idealisation of the traditional way of life that Seton-Watson discussed. The manifesto then states that "current German and European asylum and refugee policies cannot be continued as in the past", and discusses ways in which immigration can be stemmed - a reaction to the externality of the migration crisis.

Furthermore, the demographics of the AfD show how its supporters and leaders conform to this model of populism. Both Alice Weidel and Alexander Gauland, leading figures within the AfD, can be considered part of the 'intelligentsia', having been an investment banker and lawyer respectively. Support for the AfD is extremely regionalised; the 2017 election was described as the "Revenge of the East" due to the AfD's strong performance in the East of Germany. These regions were poorer, and less productive than Western Germany, suggesting that the AfD's support base can be found in the lower socioeconomic classes, further suggesting that the AfD is a populist party because its support base is found amongst the lower socioeconomic classes.

Above all else, those seeking to discover whether or not the AfD is a populist party should look no further than its preamble, setting out a definition of the party as:

"A true political alternative"

... "not presented by the political class"

On the basis of this evidence, it is clear that the AfD is populist in both outlook and makeup. Its commitment to a traditional family unit, shunning of immigration and call for a "true political alternative" all are fundamentally populist policies, giving credence to the argument that there has been a "populist surge" in German politics. Furthermore, this populist surge can be assessed as having a hugely destabilising effect on German politics for two reasons: the seats won in the Bundestag by the AfD, and the political narrative shift that the AfD gaining mainstream support led to.

In the 2017 federal election the AfD won 23% of the seats available in the Bundestag, whilst the largest party, the CDU/CSU alliance, only won 41% of all seats - 9% short of an overall majority. This led the CDU/CSU into a second 'Grand Coalition' with the SPD, making Chancellor Merkel's position increasingly tenuous as she was forced to appease different parties, with different ideologies, in order to maintain her grip on power. This difficulty manifested itself in the July immigration deal, in which Merkel promised the CSU strict border controls with 'transit centres' to send migrants back to their point of first contact with the EU. The SPD refused to accept this deal, leading to Merkel having to compromise further, agreeing that 'transit centres' would just be local police centres for processing. At the time of writing, this situation between competing CSU and SPD interests has not been resolved - with neither party willing to agree on a border solution.

The narrative shift that the AfD caused was primarily based around the concept of border controls. Before the rise of the AfD in the lead up to the 2017 election, parties were fairly homogeneous in their migrant policy, agreeing that limits were needed, whilst being broadly happy with the current situation. However, as the AfD made inroads in all areas of Germany, but particularly the East, parties had to adapt their message in order to keep appealing to voters. The most prominent case of this is the CSU, who has since pushed for tighter immigration controls, in the hope of defending seats in the October Bavarian state elections. This narrative shift, which many voters stated was the reason why they voted for the AfD, has destabilised German politics due to its potential for alienation as Germany increasingly swings to the right there is less centre ground than before - with even the SPD calling for limits on immigration. One politician told migrants "for you there is no future and no home in Germany", leaving open the very real possibility of a group to grow inside Germany that feels isolated, alienated and vulnerable as mainstream politics increasingly turns against migrants. This can only lead to destabilisation as future push backs, both politically and socially, i.e. through protest and violence, may occur.

From the above, it appears that of all the factors identified, the surge of the populist AfD is most responsible for destabilising German politics, due to both its weakening of mainstream parties in the Bundestag, and the right-wing narrative shift it promoted. However, the extent to which this populist surge is directly caused by the migrant crisis is debatable, as longer term political trends can also be seen to play a role:

As argued by Cas Mudde in the Popular Zeitgeist, the roots of modern political dissatisfaction can be found as far back as the 1960s, when traditional political parties across Europe found their support base dwindling. The once-dependable working class and religious voters no longer voted upon such partisan lines as before, whilst new ideas of neoliberalism and neoconservatism redefined what it meant to be a political party. Out of this partisan dealignment grew what came to be termed 'TINA' politics - There Is No Alternative policies - which broadly claimed that free markets, free trade, capitalist globalisation and democracy were the best, and only, way for a modern society to be run. In Britain this took the form of New Labour under Tony Blair, mirrored by Chancellor Gerhard Schröder's Neue Mitte (New centre) in Germany. This, more than anything else, led to the destabilization of Germany politics, and the rise of populist parties. Parties became increasingly homogeneous as the acceptance of the single, 'correct', way to run a country became mainstream, whilst voters became disenfranchised as they were told that their views on topics as diverse as abortion, the role of gender and immigration had no place in modern political discourse.

Above and beyond any other factor, this best explains why there has been a populist surge leading to political destabilisation in Germany. It was felt by too many voters that their opinions were not being discussed in mainstream politics, so when an party like the AfD began expressing views that 49% of the country agreed were more along their line of thinking in the current political situation, it shouldn't be surprising that its support grew rapidly. Ultimately, the surge in populism in German politics was sparked by the migrant crisis, but without underly dissatisfaction with the political system resulting from decades of TINA politics, and a lack of trust in Merkel's government resulting from the handling of the Eurozone crisis, this spark would have had nothing to light.

conclusion, there has been a definite In destabilisation of German politics since the events of the 2015 migrant crisis. This is best expressed through the loss in seats for mainstream parties in the 2017 election, but also through the internal divisions between the CDU and CSU, as well as the difficulties Angela Merkel has had holding her "Grand Coalition" together. This is due to a number of factors, including but not limited to the German government's response to the Eurozone debt crisis, German scepticism about the Merkel's 'open doors' policy and underlying dissatisfaction with the modern political system. Of these factors, the latter two can be identified as being rooted in a 'populist surge', with the rise in support for the AfD, a decidedly populist party, being due to dissatisfaction with the homogenous political elite being catalysed in the migrant crisis, leading to political instability. However, when answering the question "To what extent is the destabilization of German politics due to a populist surge resulting from the European Migrant Crisis?", we can see that ultimately, the destabilisation of German politics was due to a populist surge, though one that was more rooted in long-term political trends than the events of the European migrant crisis.

Modelling and predicting a New York City taxi trip duration.

Mikel Bober-Irizar



1. DEFINITION

1.1 Project Overview & Introduction

Taxis are a vital part of the ecosystem of many cities. In New York City alone, over 300,000 yellow cab trips are taken every day, in addition to over 500,000 ridesharing trips [1]. Due to the advent of ridesharing apps such as Uber, the total number of trips taken has almost doubled in the last two years.

To be able to operate a taxi service at such a large scale, companies use electronic dispatching systems to efficiently assign cabs to customers and spread out cabs across cities where they might be needed in order to maximize the number of rides each cab can take in a day.

One vital aspect of such systems is to predict how long a taxi trip will take once a cab is taken, so the system can understand how long a specific cab will be removed from the pool of free cabs and when it will be free to pick up more passengers. This allows dispatchers (and automated dispatch systems) to reduce passenger waiting times and increase revenue from each cab, creating a win-win situation.

<u>Kaggle</u>, the well-known competitive machine learning platform has hosted several competitions to predict the duration of taxi trips, such as the ECML/PKDD 2015 competition [2] which used data collected from taxi trips in Porto, Portugal. In order to build an algorithm that can accurately predict the duration of a taxi cab trip, I will be using the dataset provided by the recently launched and ongoing <u>New York City Taxi Trip Duration [3]</u> competition, which is a very large dataset released by the NYC Taxi and Limousine Commission.

1.2 Problem Statement

The Kaggle dataset consists of data collected from New York City over a period of 2009 to 2016 – in total, it contains information from over two million rides.

The goal of the problem is to predict the duration in seconds that a given taxi trip will take (known as the target variable) – making it a supervised regression problem. This means that the goal of the model will be to predict a continuous value, by learning from a dataset of known inputs and outputs.

This project will focus on trying to solve this problem and getting a maximal score on the Kaggle leaderboard based on the metrics described in the following section.

As inputs, we are given information such as the company which runs the taxi, the starting and ending locations, and date/time information – this is the information that we need to predict ride duration from. As the data given is tabular in nature, my approach will be to use standard supervised regression algorithms such as decision trees, support vector machines or linear regression.

Overall, I will approach the problem in multiple steps. Firstly, I will explore and visualize the data to gain an understanding of the features and where the 'signal' in the dataset lies. This will allow me to go onto feature preprocessing, where I will convert the features into formats more suited towards the regressors I will be using, as well as performing 'feature engineering', a term commonly used on Kaggle which refers to creating entirely new features which may be more predictive out of the existing features.

After I have created my final feature space that I am happy with, I will build a regressor upon the data to obtain a score. When I have obtained my classifier, I will perform parameter tuning to maximize the score obtained on a validation set, and then use the best model to create final predictions which I will then upload to the Kaggle leaderboard to obtain my final score.

I anticipate that my final solution will consist of a feature processing pipeline followed by a single trained supervised model which outputs predictions for the test set that can be uploaded to the Kaggle leaderboard.

1.3 Evaluation metrics

To measure the performance of the model, I will use the Root Mean Squared Logarithmic Error metric (referred to as RMSLE).

The RMSLE is defined on the <u>Kaggle evaluation</u> page [3] as:

$$\epsilon = \sqrt{rac{1}{n}\sum_{i=1}^n (\log(p_i+1) - \log(a_i+1))^2}$$

Where:

N is the number of samples,
p_i is the prediction of the trip duration,
a_i is the actual duration
log(x) is the natural logarithm of x.

This metric is **identical to Root Mean Squared Error** (**RMSE**), which is widely used for evaluating solutions to regression problems, **except that** the "log1p" of both the target and predicted values are taken beforehand. log1p(x) is defined as log(x + 1) - the +1 is there to avoid taking the log of 0 (which is undefined).

Most regression algorithms are designed to optimise RMSE out of the box, which doesn't necessarily mean that they will find the optimal solution for minimising RMSLE. Luckily, by training the model to predict log(trip_time) instead of the actual trip time, and then simply taking the exponential of the model's output to reverse the transformation, any model that optimises RMSE can be made to optimise RMSLE. This is because optimising RMSE on the log of the targets is equivalent to optimising RMSLE on the actual targets themselves.

There are two reasons I have decided to use this metric. Firstly, this is the official metric that we need to optimize for the Kaggle competition and the metric upon which the submissions to the competition are ranked, so it makes sense to also try to optimize this locally.

In addition, I believe using the log-error also makes more sense than directly using the error for each trip. This is because we care more about the error in each sample **relative to the trip time** as opposed to simply the absolute error.

For example, one would say that a 5-minute trip incorrectly predicted as 2 minutes is much worse than a 50-minute trip incorrectly predicted as 47 minutes. If the RMSE metric was used, these two trips would have the same error. However, RMSLE would penalize the 5-minute trip more, even though both predictions were incorrect by three minutes. This seems more reasonable to me than penalizing them both equally, so for this reason I think RMSLE better represents what we actually want the model to minimise.

2. ANALYSIS

2.1. Data Exploration

As part of the dataset from Kaggle, we are given two files, a train.csv and a test.csv. These two files represent the training and testing data given by Kaggle – the formats of the files are identical except for the fact that the testing data does not have target values included.

The training set contains 1,458,664 trip records, while the test set contains 625,134 records, making this a rather large dataset. Each trip record is represented by a row in the csv file, and has several features given for it. I have described each feature given briefly on the following page.

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Feature	Description
id	The ID of the trip. Not to be trained on
vendor_id	A categorical variable indicating the taxi provider associated with the record
pickup_datetime	The date and time that the taxi meter was engaged (and the passenger was picked up)
dropoff_datetime	The date and time that the taxi meter was disengaged. This feature is only present in the training set.
passenger_count	The number of passengers in the vehicle
pickup_longitude	The longitude of the passenger pickup location as a float
pickup_latitude	The latitude of the passenger pickup location as a float
dropoff_longitude	The longitude of the passenger's destination as a float
dropoff_latitude	The latitude of the passenger's destination as a float
store_and_fwd_flag	Whether the trip was "store and forward", meaning that the vehicle did not have any connection to the server during the trip and trip details were uploaded later. Denoted by "Y" or "N" values
trip_duration	The total duration of the trip in seconds. This feature only appears in the training set and is the target value .

This means we have a total of 8 features that we can train on, which includes one categorical, one Boolean and one timestamp feature, the rest being float-valued.

From the statistics above we can see that the (latitude, longitude) pairs are clustered in a very

	passenger_count	pickup_longitude	pickup_latitude	dropoff_longitude	dropoff_latitude	trip_duration
count	1458644.000	1458644.000	1458644.000	1458644.000	1458644.000	1458644.000
mean	1.665	-73.973	40.751	-73.973	40.752	959.492
std	1.314	0.071	0.033	0.071	0.036	5237.432
min	0.000	-121.933	34.360	-121.933	32.181	1.000
25%	1.000	-73.992	40.737	-73.991	40.736	397.000
50%	1.000	-73.982	40.754	-73.980	40.755	662.000
75%	2.000	-73.967	40.768	-73.963	40.770	1075.000
max	9.000	-61.336	51.881	-61.336	43.921	3526282.000

small region around (-74, 40) – this is expected since that is the location of New York. However, there are some very large outliers thousands of miles away – these are most likely GPS errors. A similar case can be seen in the trip_duration target variable: The max of this variable is equal to just over 40 days, which is obviously an erroneous measurement. Such outliers are to be expected in such a large dataset. However, they are not very common – for example, there are only 82 samples out of 1.5 million which have abnormally low longitude values, so these anomalies are likely to be ignored by most machine learning algorithms.

The categorical vendor_id variable contains only two possible IDs, with 47% of the taxi trips containing the vendor ID '1' and the rest containing '2' – this variable should instead be treated as a simple boolean variable (one with two possible values, which can be mapped to true/false). The store_and_fwd_flag variable is also a boolean, but is only true in 0.5% of taxi trips, showing a rare event. It's unclear how this variable could affect ride times.

2.2 Data visualization

In this section, I will visualize and analyse some interesting aspects of the dataset given to gain a better understanding of different aspects of the data. The information gained from this is vital in order to figure out what feature engineering may help extract signal from the data, as well as what types of models may work well on the data.

I hypothesise that this problem has a substantial aspect of time-dependence, meaning that the distribution of the target changes a lot with respect to the time of day (traffic conditions may change, for example), as well as what day of the week or year it is (special occasions may affect taxi trips, for example). For this reason, I think it is important to investigate how the pickup datetime affects the data. One important aspect is to look at how the split between the training and testing sets are done. There are two different ways this could have been done:

- The testing data is sampled from the same period of time as the training data. This would allow us to get a lot more information out of the training set – for example, when predicting on a sample in the test set, we could look at trips made around the same period in the training set to understand the current traffic situation.
- 2) The testing data is sampled from a period of time in the future (relative to the training data). This creates a very different problem: instead of trying to predict a subset of the taxi rides around the same time, the challenge is instead about creating a model which could predict taxi trip time on a future day (where the outcome of recent taxi trips is not known). This option would make more sense for the data, as taxi companies are not interested in predicting on past data but instead obtaining a model that can tell them how long future rides will take.



From the above plot, we can observe that the dataset is relatively uniformly (apart from an unusual dip in in January) distributed throughout the first half of 2016 (183 days in total). More interestingly however, both the training and test sets occupy the same distribution, which shows that the first approach described above was taken in preparing the data. This means that we are actually given target values for other taxi trips around the same time period of each test trip, and these can be used to better predict the trip time.

We can also look at how the trip duration changes based on the time of day using a boxplot:



This graph shows that the duration of taxi trips varies quite dramatically based on the time of day. In the early morning (5-8am) and to a lesser extent in the evening (19-21), customers tend to take quicker trips than during other parts of the day. At 8am and onwards, the average trip duration tends to get longer. I believe this is due to increased traffic conditions at peak times meaning that taxis cannot travel as fast. However, to rule out the possibility of people simply travelling further during the day, I need to analyse the speed of the taxis directly.

Since I am only given co-ordinates of the pickup and dropoff points, we are not given any information about the speed of the car. However, this can be approximated and inferred from other information: For this analysis, I will use the Manhattan (L1) distance between the two points and divide this by the trip duration to get a rough estimate of average speed. If we then plot the calculated speed using another boxplot, more interesting observations arise.



Here, the effect is even more pronounced. We can see that by my speed metric, the early morning is the time at which taxis travel fastest, and this slows down very substantially (average speed roughly halves) beginning at around 7am onwards. In fact, we can see that these observations almost perfectly match <u>data released by the NYC Taxi Commission in</u> <u>2013 [4]</u>, which put the peak speed at 5:18AM and showed the same relationship for the rest of the day.

From the above analysis, I can see that time has a very large effect on taxi trip duration. For this reason, I will focus on building features that can capture this relationship (as well as the relationship with taxi speed) in the modelling phase, which should help predict trip duration at different times of day better.

In addition, we can look at the distribution of the target variable to understand what we are trying to predict:



From this, we can see that taking the log of the target value makes it much more normallydistributed. This should make it easier to be modelled by a regressor versus the long-tailed distribution of the original target value (where very large outliers could cause the model to learn to overestimate).

2.3 Algorithms and Techniques

XGBoost

For training models, I have decided to use the <u>XGBoost</u> algorithm [5]. XGBoost is an implementation of a machine learning algorithm known as Gradient Boosted Trees (GBT), also referred to as gradient boosting machines (GBMs). The concept of this algorithm is as follows:

- Fit a decision tree to the data. The model constructs a decision tree by exhaustively looking for the splits (eg. hour of day > 6) which are the best predictors of trip duration. Then, each leaf of the decision tree is assigned a value equal to the mean of all the trips in the training dataset which falls into that bin.
- 2) Evaluate the model against each data point in the training set.
- For each sample in the dataset, increase its weight/importance based on how incorrect the model was in the last step,
- 4) Fit another decision tree to the data using the reweighted data, which tries to predict the residual (the difference between the current model and the actual values) – this new tree will fit more to the areas where the previous trees did not perform well.
- 5) Redefine the model as the sum of all the trained decision trees, and go back to step 2. By training multiple decision trees, and summing their results, the outlier results of each tree are cancelled out by the other trees, increasing the robustness of the model. This works much like how taking multiple measurements and averaging increases the precision of scientific measurements, and is generally known as an ensemble method.

The result of this is an algorithm which has the benefits of other tree ensemble models (such as random forests) while outperforming other similar techniques in performance.

I have chosen to use this algorithm for multiple reasons:

- Since it is a tree-based algorithm, it is completely scale-invariant. This means it can very easily handle features with very large or very small values, such as latitude and longitude, reducing the amount of preprocessing that needs to be done. Other machine learning algorithms do not behave

well when the different features have different distributions.

- Also due to its tree-based nature, it can learn non-linear relationships as well as relationships between features. This allows it to directly use the latitude and longitude features – it can learn different patterns for different areas of New York by splitting on these features where a linear model would not be able to. For example, using four splits in a branch the model could check if the car is located in a specific rectangle of the latitude and longitude.
- XGBoost is widely used in (and often wins) Kaggle competitions as it is known to tend to outperform other models with a range of complex non-linear features, as we are given here.

The downside of using XGBoost is that it can potentially be prone to overfitting due to its reps, where the model begins to learn slight patterns in the dataset that only exist due to noisy data, reducing its performance when predicting on new data. However, this is much less of an issue when large datasets are present (as is the case here), and this can also be mitigated by using a validation set, which the model is not trained on, in order to evaluate its performance. The model can be evaluated on the validation set after every tree is added and 'early stopping' can be used to stop training the model when it stops improving on the validation set, lest it overfit.

When training, XGBoost takes in a matrix of shape (samples, features) and a vector of target values, returning a model. This model can then be used on another matrix of the same format (the test set) to return a vector of predicted target values. XGBoost only supports numerical input features, so features such as datetime and categorical variables will need to be transformed beforehand. Formally, and at a high level, XGBoost builds a model out of a set of additive functions to predict the target y as a function of the input space x:

$$\hat{y}_i = \varphi(x_i) = \sum_{k=1}^{K} f_k(x_i) \quad f_k \in \mathcal{F}$$

Where \mathcal{F} is the space of (CART) decision trees [6] representing mappings from the multidimensional input space to a single real output value $\mathcal{F}: \mathbb{R}^N \to \mathbb{R}$, φ represents the learnt model, and \hat{y}_i is an estimate of the target value y at row i.

The goal is to minimise the sum of squared errors between the model's output and the true target values.

$$min\sum_{i}(y_i-\varphi(x_i))^2$$

To do this, each tree is built to minimise this loss function when summed with all the previous trees:

$$L(a,b) = (a-b)^2$$
$$f_t = \underset{f \in F}{\operatorname{argmin}} \sum_i L(y_i, \ \hat{y}_i^{(t-1)} + f(x_i))$$

Where f_t represents the tree learnt at iteration t, and $\hat{y}_i^{(t-1)}$ represents the sum of all previous trees (the current output of the model). In practice, since the loss function is convex and twice differentiable, f is optimised through second-order gradient descent, using several tricks to help convergence and improve the model's performance (which will not be covered here). [5]



There are a few basic XGBoost parameters that need to be tuned to obtain optimal results:

Parameter	Def	ault	Explanation
max_depth		6	The maximum depth (number of splits in a branch) of each individual tree in the ensemble.
colsample_bylev	el	1.0	The proportion of input features which are randomly selected and available to the model to use at each level of the tree. Decreasing this means trees are more likely to be different, which can improve ensemble performance.
subsample		1.0	The proportion of the data which each tree is trained on. Decreasing this increases the difference between trees, which works for the same reason as colsample_bylevel.
min_child_weigh	t	1.0	The minimum amount of "weight" a tree node must have to be included. "weight" loosely means how much it contributes to the model. Increasing this can help stop the model
eta		0.3	The learning rate (amount of reweighing done in- between trees). Decreasing this always helps performance, but learning takes much longer for diminishing returns.

2.4 Benchmark

Arguably the simplest benchmark which can be used to compare between models is the performance obtained by always predicting the same value (this can be considered equivalent to a prior probability). In this case, I will find a single trip duration that minimizes RMSLE on the training set, and then use this to create a submission on the Kaggle leaderboard.

The optimal trip duration estimate can be found with:

optimal trip duration = expm1(mean(log1p(trip durations)))

This yields us a trip duration estimate of **642.54 seconds**, which gives us a **0.796 RMSLE score** on the training set and a **0.798** score on the Kaggle public leaderboard.

Thus, we can say that for a model to have learnt anything less trivial than the prior of the data, it must have a RMSLE error of less than our benchmark **0.798** on the leaderboard.

3. METHODOLOGY

3.1 Data Preprocessing

To convert the data into a format which XGBoost can effectively use, as only numerical features are supported, some of our input features must first be transformed.

store_and_fwd_flag

The first feature that needs to transformed is the store_and_fwd_flag. This feature is a boolean, comprising of the possible values "Y" and "N". To transform this feature, I simply map "Y" to a 1 and "N" to a 0, which means it can be digested by XGBoost.

pickup_datetime

This feature is slightly more complicated to preprocess. It is given as a string in the form of "30/06/2016 23:59:58". To preprocess this feature, I do several transformations resulting a set of new features based on this time. First, I use the pandas package to parse the string into a Timestamp object from which I can extract useful features. From this data, I extract the following features:

The Unix epoch time (number of seconds passed since 1970): This is useful for the model as it allows the model to learn patterns over longer periods of time in the dataset (for example, some seasonal effect, or a gradual decrease/increase in speed over the period of the dataset).

The minute of the day: This is calculated using [hour*60+minute_of_hour] and is useful because it allows the regressor to model daily speed patterns (such as how the taxis move fastest at 5am).

The day of the week: Represented by an integer, this feature allows the regressor to model different patterns during different days of the week – I expect traffic conditions are substantially different on weekends, so this allows the model to distinguish between conditions on weekdays and weekends.

vendor_id

Although this feature is technically a categorical feature, it does not require preprocessing, such as one-hotting to create a separate feature for each vendor. This is because there are only two vendors, so the feature can be treated as a boolean feature representing "vendor 1 vs not vendor 1", as discovered in the Data Exploration section.

Dropped features

I have decided to drop a few features altogether from the dataset. The id variable simply identifies the row and does not provide any information relevant to the trip, so I have removed this feature from training. The dropoff_datetime variable has also been removed because it is only present in the training set, and does not provide any new information as it is simply equal to pickup_time + trip_duration.

FEATURE ENGINEERING

Another important part of my approach is "feature engineering" – the practice of making new features that allow the algorithm to model aspects of the data it could not easily model before. Tree-based models such as XGBoost are unable to model mathematical operations such as addition and subtraction between variables. This means some signal in the data which could provide a large boost to performance simply cannot be modelled: for this reason, it is important to compute these helpful features in the preprocessing phase.

One feature that should help regression but cannot easily be deduced by the model is the distance between the pickup and dropoff locations.

Distance

There are several ways to measure distance between (longitude, latitude) pairs. I have opted to use several methods and provide all the distance metrics to the model so it can use whichever one is most predictive (or a combination of several).

The first is the L1 distance, also known as the Manhattan distance – this is the distance while only being able to move north/south and west/east. This distance could be useful because it resembles the road system of blocks in NYC, where cars can't travel diagonally between destinations. It is defined as:

$$d(a,b) = |a_{lon} - b_{lon}| + |a_{lat} - b_{lat}|$$

The next metric is the L2 or Euclidean distance. This is the direct distance between two locations, when travelling as the crow flies:

$$d(a,b) = \sqrt{(a_{lon} - b_{lon})^2 + (a_{lat} - b_{lat})^2}$$



Because the world is a sphere, this means that calculating the Euclidean distance between latitude and longitudes (which assumes the world is flat) is not always an accurate measure of true distance between two points.

Because of this, another more complicated distance formula known as <u>great circle distance [7]</u> (implemented by the <u>haversine formula</u>) was created which calculates the distance between two points following the curvature of the earth's surface. Because of this, the Euclidean distance can be thought of as the 'map distance' while the haversine distance can be thought of as the real distance. At such small scale, the difference should be minimal but I have decided to include both into my calculations.

Using this, I create three new features: dist_l1, dist_l2 and dist_haversine.

Direction of travel

I hypothesize that knowing the direction of travel can also be helpful to the model – for example, there may be less traffic in an area when going north versus when going south. Because we're not directly given the route that the taxi takes, we cannot know exactly which direction the taxi is taking, however, we can make some approximations.

Because of this, I created two new features, delta_lon and delta_lat, which are simply equal to the starting lat/lon minus the destination lat/lon. These features quantify how far west or east the taxi travelled, as well as how far north or south it travelled.

In addition to these features, I also calculate the bearing angle between the two co-ordinates. This process is a little more involved. Thankfully, numpy implements an <u>arctan2</u> function, which can easily calculate the bearing (in radians) of a single vector from the x-axis. This can be adapted to calculate the bearing between two points (our pickup and dropoff co-ordinates):

$$angle(a,b) = \frac{180}{\pi} \cdot \arctan(b_{lat} - a_{lat}, b_{lon} - a_{lon}) + 180$$

As an example of the output of this angle feature, if the destination is directly east of the pickup, the angle will be 90°, while if the destination is north east, the angle will be 45°. This feature allows the model to better understand what direction the cab is going to travel in from its pickup.

Current traffic conditions

Because we are given information about past trips that have occurred in the dataset (and other trips around the same time), we can see how busy the taxi service is at the time of a given trip, and potentially use this to infer the traffic conditions at the time. The basic idea is that the number of taxis recently taken reflects the total number of cars currently on the road – at busier periods more taxis will be taken. By incorporating this info, the model can learn information about conditions beyond basic repeated trends (for example, holidays or special events may not fit the normal daily traffic and by looking for abnormal taxi usage the model can spot times when trip times may be unusual.)

To capture this information, I create two new features, daily_count and hourly_count. These features represent the total number of taxi trips taken during the current day and current hour respectively. To calculate this, I partition the data into days and hours and count the number of rows in each partition, adding the count as a feature to each partition.

Time estimates based on speed of other taxis

Since the training and testing sets cover the same time period, this means that for any given point in time in the dataset, we know the trip duration for ~75% of the taxi trips. Using this trip duration as well as the approximate trip distance calculated earlier, we can work out the average speed of each taxi. Then, by assuming the test set taxis travel at the same speed, we can use this to create time estimates based on current conditions at any given point in the dataset.

Using this principle, I create a feature called haversine_speed_estim, which is calculated with the following process:

 For every sample in the training set, I compute the haversine distance, and divide this by the trip_duration (the time taken to move that distance) to obtain the average speed of trip.

- For every hour in the dataset, I take the mean of the speed across all the training samples, calculating the average taxi speed for that hour.
- 3) For every sample, I calculate the haversine distance travelled, and multiply it by the average speed for that hour to obtain an estimate for trip_duration. This can be performed even on the test set, giving me time estimates for the entire dataset that can be trained on with XGBoost.

All code for the implementation is available at https://github.com/mxbi/mlnd-capstone!

3.2 REFINEMENT

Running my initial implementation as described above converged after approximately 1,200 rounds of boosting, which when submitted to the Kaggle leaderboard, gave me a score of 0.39098 RMSLE on the leaderboard, putting me in 167th position at the time of writing. After obtaining this result, I went on to refine my model.

To improve the model, I decided to opt for a parameter tuning approach to find the set of parameters that best optimizes the RMSLE on the validation set, and hopefully that which also optimizes the RMSLE on the Kaggle leaderboard. Usually, I would use a Grid Search approach over a space of predefined parameters for this. However, training the model is slow due to a combination of the resource-intensive XGBoost algorithm, a very large training set, and many features (up to 30 minutes on a 16-thread Intel i7). This means that a grid search approach is unfeasible for this problem.

Instead I have decided to use my own approach to parameter optimization which has worked well for me in the past and is much faster than grid search. The basic idea is the following:

- Some reasonable guesses are used as default values
- 2) The parameters are tuned one-by-one. For every parameter:
- Train a model for every value of this parameter to be tested, using the default values for all other parameters.
- 4) Find the value for this parameter that gave the best score
- 5) Update the default value for this parameter using the found best value
- 6) Move to optimizing the next parameter

This means the optimization problem is linear with the number of parameters, while a grid search (trying every possible combination of values) requires exponential computation with the number of parameters, making it much faster.

I have defined the following search space for my parameter optimization:

Feature	Values
max_depth	[4, 5, 6, 8, 10, 12, 14]
colsample_bylevel	[1, 0.9, 0.8, 0.7]
subsample	[1, 0.9, 0.8, 0.7]
min_child_weight	[0.5, 1, 2, 3]

This required 19 models to be built to find the optimal combination (versus 448 with grid search). The following results (on the next page) were obtained as a result of the optimization.



The full code for this optimization is in model_search.py. From this, an optimal set of parameters were obtained:

max_depth = 12, colsample_bylevel=0.7, subsample=0.99, min_child_weight=0.5

In creating my refined model, I trained another XGBoost model using these tuned parameters. In addition, I also halved the eta (learning rate) to 0.05, which leads to slower convergence but eventually nets a higher score. The code for this model is available in model_optim.py.

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4. RESULTS

4.1 MODEL EVALUATION AND VALIDATION

The final model as implemented in the refinement section gave a big improvement on the leaderboard over my initial implementation, improving to **0.38092 RMSLE** from 0.39098 in the initial model. This improved my rank 99 places, leaving me at 68th out of 767 teams.

While the raw RMSLE result I obtained is satisfactory, I would like to verify whether the model is robust and can adapt across the whole training set or whether it only generalizes to a subset of data. To begin with, I will analyse how correct the model predictions are depending on what the target variable is, to check whether the model is able to predict well for both short and long trips:



From these plots, we can see a few interesting observations. First off all, we see that for most trips across most durations, the XGBoost model is reliable in generating good predictions. However, on the log plot on the right, we can also see that for samples where the log duration is less than 5, the model is unable to predict these very accurately on the log-scale. This corresponds to two-minute rides or lower, so it makes sense that these would be difficult to predict accurate to the second, as each trip has an inherent random time added to it (getting in the taxi, paying, etc).

However, looking at the graph of real time on the left, we can see that for the most part the model still predicts the trip time well – there are almost no cases where XGBoost overpredicts the trip time, but a few cases where it underpredicts. I hypothesise this is because of some effect on the taxi which is





not captured in any of my features (e.g. passenger wants to take a route via some other location).

We can also look at how well the trip duration estimation works during different times of day (top of page).

The above plot shows the effect that differing times of day has on the performance of my model. The model appears to perform worse during the early hours of the morning when speed is generally faster and perform better when there is lots of traffic – this could have something to do with the model's worse performance (on a log-scale) predicting short trips. However, while I would argue this trend in the model performance definitely exists, the performance of the model is satisfactory at all times of day.

From this analysis, I am relatively confident that the model is robust across different rides and can be trusted.

4.2 Justification

My final solution greatly outperforms the benchmark solution, more than halving the error, going from a **0.798 RMSLE** score down to **0.381**. Another metric which can be used to compare these two solutions is to look at how often they predict trip durations close to the actual durations.

If we define a good prediction as being less than 20% out from the actual (absolute, not log) duration, we can calculate that while the benchmark solution is correct just **21%** of the time, my refined model brings this value to **59%**, showing the extent of the improvement.

I feel that the final solution I have presented here has been able to well capture the signal from the data in a way significant enough to produce a useful model. Looking at the scatterplots of predictions in the last section, it can clearly be seen that the model has learned to well-predict the duration.

5. CONCLUSION

5.1 MODEL ANALYSIS



This plot displays XGBoost's built-in feature importance metric, displaying how much each feature contributed to the final solution. Surprisingly, the pickup longitude and latitude are the most important singular features in the dataset – this may suggest that XGBoost actually learnt a very granular speed pattern over the pickup location. From this plot, we can also see that L1 (Manhattan) distance is the most important distance metric for this problem. While all three distance metrics are lower in the feature importance than I would expect, it is worth noting that the features are lower in the importance as XGBoost has been given the similar information multiple times, so it does not rely too much on a single distance feature: the combined importance of these distance features would rank at the top, especially if you added features such as delta_lat and delta_lon.

Interestingly, store_and_fwd_flag provided basically no information to the model. This makes sense, as I did not see how this feature would impact trip duration during data exploration.

5.2 Reflection

My final end-to-end solution can be described in a few simple steps:

- First, the input data is preprocessed to remove or transform non-numerical features.
- Then, new features are created based on existing ones, in order to bring out additional signal from the dataset
- A parameter-optimized XGBoost model is trained on the newly engineered data to build a model that can predict the duration of a taxi trip
- This model can then be used on new input trip data to create a prediction about a trip duration

I think the most interesting part of this project was the feature engineering – this is where, instead of going through the same (almost muscle memory) technique and processes as in other problems, experimentation and new ideas are required to come up with novel features. I also found it the most challenging, as some of the features I built were complicated to implement in an efficient way, for example the time estimate features.

I think my final solution does fit what I expected to have for this problem. However, while this model performs well in the Kaggle leaderboard, I do not feel it would be an appropriate model for use in an actual taxi trip prediction situation. This is because in this dataset, the training and testing sets are aligned in time, meaning that you are given ground truth for data in the same time period as the test set. In a real-world situation, this data would not exist – you would only have data on *past* taxi rides, and not data on current or future rides. Because of this, the model would need to be retrained with a test set which is forward in time, and features such as the time estimate feature would not be possible in its current form as the ground truth data it is based on would simply not be available.

5.3 Improvement

I feel that there are several aspects of my solution that could be improved, but I did not have the time to investigate these aspects for my project.

Firstly, I think that there is a lot more scope in the sort of feature engineering that could be done, especially incorporating outside data. For example, collecting and using weather data as a feature or a list of holidays and special occasions as features could have provided the model with more info that could let it pick up patterns in traffic conditions. In addition, I think that my approximate methods of measuring distance could be replaced, for example, with a much more accurate estimate based on a route plotted on mapping data. However, this is not something I could implement myself.

In addition, I feel the modelling side could also be improved. Very often, Kaggle solutions are made up of ensembles of large numbers of models whose predictions are combined (thus combining the strengths of each model), resulting in a meta-model with much higher performance on the leaderboard than any single model. I think this would be the way forward if I wanted to improve my submission.

An improvement on the score I have set in this report is definitely possible. At the time of writing, the highest achieved score was **0.36531 RMSLE**, a large improvement. One of the top competitors shared details about his solution and said that it used an ensemble of models on top of 59 engineered features, showing that a large amount of work could be done to further improve my score.



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Method in the madness: using religion to decipher Olivier Messian's music

Sam Jones

Pierre Boulez, one of Messigen's students, once criticised his teacher's work as "Music of the brothel". This description rings true with many music critics internationally who believe Messiaen's compositions to be a crude stirring pot of ludicrous invented harmonies and stolen birdsong. However, in my opinion, this composer can't be judged in the same way as his 20th century counterparts. I believe his Catholic religion was more than a justification for his divisive works and that with a spiritual presence in mind these works can make musical sense. I therefore ask the question of whether Olivier Messiaen's music is just refusal of good taste, or whether it has intriguing intricacies that listeners need to appreciate in order the see the whole stainedglass window.

In an interview with Bernard Gavoty, Messiaen replied to a question about his faith by saying: "the first idea I want to express is the existence of the truths of the catholic faith". As a result, musicologists studying the works of Messiaen can be sure that his music is built around religious elements, rather than them just being a mere afterthought. Nonetheless, the problem arising from this statement is the question of what these truths of the catholic faith are. Throughout music history before Messiaen, religious music was found predominantly in the liturgical setting of the church. Employees of religious institutions such as Bach composed cantatas and large-scale masses to often glorify texts they had been provided with or specific tangible aspects of religion such as the Trinity. A notable distinction must therefore be made between Messiaen and his predecessors; just a glance at his oeuvre affirms Messiaen did not write religious works for the church and neither did he write religious works simply to explain the meaning of a text behind it. The reason for the above is that Messiaen wrote every single note with a vivid image attached to it, one will not hear symbolism of the Trinity in his works but instead his sentiments to God directlythose of awe, fear and joy.

There are small, obvious pointers to spirituality in Messiaen's music such as the nine movements in *La Nativité* symbolising the nine months of Mary's pregnancy and the liturgical titles given to works in his organ cycles. However, the deeper feelings about God that Messiaen wanted to depict are the qualities of timelessness and dazzlement. The sensation of timelessness is seemingly contrary to the definition of music being sound waves heard across time; but it is a core component of Messiaen's composition style and of his religious beliefs, proven by a verse from the Book of Revelation Chapter 8: "I am the Alpha and the Omega, says the Lord God, who is and who was and who is to come "representing a God with no beginning and no end. The eternality of his lord is displayed in titles of his music to start: The Turangalîla Symphony's unusual title can be split into two Sanskrit words of 'Turanga' meaning time and 'Lîla' meaning play, playing with time therefore has the obvious connotations of timelessness. Symmetry is Messiaen's music can also represent something without distinct beginning or end (perhaps because the beginning and end are connected so strongly). We can study the movements of La Nativité and imagine symmetry in the energy or significance each movement has. In this way, the first 4 movements can be viewed as a build-up of energy before the pivot or climax in the 5th movement before gradual decreasing levels of energy in the closing 4 movements. The energy I refer to here is not inherent in the music itself, meaning that the middle movement is not louder or livelier than the fourth but the significance of the message it carries is greater for Messiaen (the message can be interpreted in part by the biblical context printed for each movement). Furthermore, the titles of each movement have symmetry themselves, there is a chiastic relationship between them where each title has a corresponding one across that middle movement. It can be best explained visually as:

1. The Virgin and the Child (La vierge et l'enfant)

2. The Shepherds (Les bergers)
3.Eternal Designs (Desseins eternels)
4. The Word (Le Verbe)

5. God's Children (Les enfants de dieu)

6. The Angels (Les Anges)

7. Jesus Suffering (Jesus Accepte la Souffrance)

8.The Wise Men (Les Mages)

9. God Among Us (Dieu parmi nous)

Take for example movements 2 and 8, which have a relationship within the Nativity story, because the departure of both the shepherds and the magi are mirrored in these movements. On a deeper level, movements 3 and 7 explore the truths of the Catholic faith in more detail; "Eternal Designs" symbolises God's predestination for us to be his adopted children but for this to be achieved Incarnation is necessary. God taking on flesh in the form of Jesus therefore assumes his suffering which is then the title of movement 7. Each pair of movements has a religious link behind it and this second symmetry also provides a sense of timelessness to portray Messiaen's religious beliefs.

Messiaen did not have an evangelical aim when composing but instead wanted listeners to have experiences of unrivalled joy and clarity which he associated with his spirituality. Under his mother's influence, he discovered a love of fairy tales from a young age and these certainly contributed to his fantastical realisations of spiritual characteristics. The results in his music are the bright, vibrant and sometimes chaotic sounds we hear in all his works, or éblouissement as he described it. Inspiration for this writing can be seen in the book of Exodus: Chapter 24, Verse 16 "The dazzling light of the Lord's presence came down on the mountain". It is widely thought that Messiaen had a form of synaesthesia (a sensory stimulus being experienced through two different sensory responses) which meant when he heard sounds he also saw colours in his head. Consequently, "dazzling" music for him may have provoked different reactions from listeners. In Error! Reference source not found., a measure from the end

of "Joie et Clarté des Corps Glorieux" (part of the organ cycle Les Corps Glorieux) is shown with a specific focus on the second bar. When heard, the listener may make the connotation with loud, blaring brass (this requires the trumpet stop on the organ) which obviously is a bright, dazzling sound. For Messiaen on the other hand, all these chords are inversions of his invented accord sur dominante -Chord of the Dominant. According to Messiaen's Technique de mon langage musicale, this is formed of every note present in an octave of a major scale. In this instance, the chord of the dominant is played with the subsequent resolution as appoggiaturas transformed into added notes (the construction of this is shown below). Every one of these chords carries a combination of colours as well (through his synaesthesia), for example the first beginning on D# he described as "burnt-earth crystals, amethyst violet, clear Prussian blue, warm reddish chestnut, with stars of gold".



Figure 1: Les Corps Glorieux, Movement 6 – "Joie et Clarté des Corps Glorieux", Measure 17

The chords in the music of Figure 1 are built in a way that Messiaen carefully describes in *Technique de mon langage musicale*. First, he takes every note from a major scale (in **Error! Reference source not found.**, this is C major)



Figure 2: Technique de mon langage musicale, Volume 2 – Exemples Musicaux, No. 201 Secondly the stave in Error! Reference source not found. shows the supposed resolution to the first chord, arriving at a jazzy G major chord with an added 7^{th} and 9^{th} .



Figure 3: Technique de mon langage musicale, Volume 2 – Exemples Musicaux, No. 202

However, Messiaen then takes another step (in **Error! Reference source not found.**) and uses the Chord of the Dominant as the consonance and lets an



ind F# in the third stave) fall to his invented chord.

Figure 4: Technique de mon langage musicale, Volume 2 – Exemples Musicaux, No. 203

Messiaen finally creates his accord sur dominante appoggiaturé by creating added notes out of the



appoggiatura and superimposing them on top of his original chord (**Error! Reference source not found**.). This example is on D# as is in Les Corps Glorieux.

Figure 5: Technique de mon langage musicale, Volume 2 – Exemples Musicaux, No. 206

On other occasions, Messiaen's condition does provide a source of religious inspiration; he saw pure white when hearing C Major, a colour which obviously has spiritual associations. At the very end of his opera Saint Francois d'Assise, he uses a C major with an added 6th chord to represent the dazzling white light of God, and the Saint's ascent into Heaven.

Another way of dazzling listeners for Messiaen was through perpetual changing their sensations between ease and unease (between the familiarity of functional harmony and more unknown territory delving into the more atonal). Below is a melodic reduction from the 5th movement of the Turangalila Symphony (Figure 6) and as one can see it is not a particularly complex idea. The rhythm is simple and regarding pitches, all the most dissonant of intervals such as the tritone are avoided. In this sense it is a standard phrase that by itself that would have sounded quite unobtrusive for listeners. However, the harmony played underneath this line could not be further from this. It is underscored by chords full of augmented 4ths and other unusual harmonic colours, meaning the audience is left in a strange environment where they feel comfortable and uncomfortable at the same time. Moreover, this line is given prominence by the mysterious Ondes Martenot, meaning the challenging sensation provided by the harmony is tamed slightly by the melody. It is this constant juxtaposition of conventional and unconventional from Messiaen that Paul Griffiths calls "stupefyingly vulgar" and is in part what makes him distinctive. On top of this there is an exciting 3/16 time signature that interacts with this harmonic action to send the listener into a sort of sub-conscious exuberance.





Messiaen himself described his attempts at achieving éblouissement when he visited Japan in November 1985. In Kvoto, he gave a lecture (now published as the Conférence de Kyoto) during which he used musical examples from La Transfiguration de Notre-Seigneur Jesus-Christ with relation to éblouissement. The work is written for seven instrumental soloists, choir and orchestra, with a structure perhaps the most closely reminiscent of Bach's passions and general 18th century oratorios of all his music. The text of the work is a jigsaw of scriptural writings from the bible and theological pieces from authors such as Thomas Aquinas which all describe an aspect of the transfiguration of Christ. The first passage Messiaen cites is in Part VIII, the third recitative starting: "Et ecce vox de nube..." where Peter, Jacob and John are stood on the summit of a mountain below a luminous cloud and experience Christ's transfiguration, hearing God say to them: "This is my beloved Son, in whom I am well pleased; hear ye him". The composer wants to convey his concept of éblouissement in three ways during this passage, especially when the words "This is my beloved Son". These are described both partially in his footnote on the score and fully during this conference. The first is

the presence of God which necessitates light. Quivering light (depicting the cloud and the Son) is provided by groups of glissandi in the strings (all different lengths and tempi) combined with trills on the triangle and chimes. The second is the voice of God, which not only requires physical voice in performance but also harmony to confirm it is God speaking. This is achieved through multicoloured chords played tremolo, the colours present in these chords change at different speeds; the Voice (according to instructions on the score) should sound distant and high up, this is achieved through a singer's position on the stage- maybe off stage. The third way is the triumph and glory of the God which is achieved through texture and harmony. The crescendo nearing the end of the passage obviously is symbolic of his glory, but also the consonant "victorious third" heard which is contrasted to the accords tournants (turning chords) played before.

The back three desks of violins play these incredibly chromatic trills to create what Messiaen called "turning chords" (Error! Reference source not found.). This dissonance is emphasized later when the interval of a third is heard.



Figure 7: 1st Violin Parts, La Transfiguration de Notre-Seigneur Jesus-Christ, Part VIII– "Récit Évangélique", Bars 39-42

The "quivering light" Messiaen was referring to when he described the presence of God is illustrated by the trills on the triangle and chimes as shown here. The intentions of the composer are made clear in his detailed analysis in the footnote.

A problem arises for Messiaen though in the communication and reception of illustrating and illuminating theological truths. God is the one being that avoids all description or explanation, so illustrating him is impossible; an issue that Messiaen completely accepts. Upon describing his *Quatuor pour la fin du Temps* he exclaimed: "this is mere trying and stammering, when imagining the devastating grandeur of the subject!". Elsewhere, he proclaims he has only made feeble attempts at the task- once saying "How to express all this with an ondes Martenot? I am completely unworthy of all this.". Although this difficulty in communication exists for Messiaen he still attempts to express his faith,

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because he said music was the only art form (and the most immaterial of them all) that could do even a small bit of justice in depicting the Faith. Musicologists have for a long time pointed out a flaw in Messiaen's repertoire, being that the overwhelming moments of music (apparently designed to provide extra-musical experiences) also repeat themselves over and over. The composer was attempting to recreate a religious breakthrough or in simpler terms, a miracle. The contradiction present is that such an extraordinary event should only take place once for it to be so miraculous; Messiaen however repeats his efforts to achieve this feat. Therefore, the content of his music surely negates the truths behind it. One must analyse the subtleties of these repetitions to understand his reasoning though, because it seems he has the concept of sembable (French idea of similarity) in his head. Every occasion a moment of éblouissement is heard in his music there is a slight variation, this is usually due to the colours experienced through synaesthesia and the context surrounding the occasion (such as the timbres of instruments present). This explanation is provided by Messiaen in the opening paragraphs of his Kyoto paper and is key to answering critics of his success in delivering religious music. Another issue arose for him in the form of reception to his works, for how was a listener supposed to relate to God when the links to him and descriptions of him were embedded so deeply within the score. At the time, he blamed this lack of appreciation on the lack of understanding for the Christian faith listeners had, for the Catholic faith was slowly becoming less popular in Europe. Messiaen blamed this demise partly on the Second Vatican Council; the Pope's changing views on transubstantiation and use of language in liturgy were certainly unpopular with Messiaen. The decision to start worship in the vernacular was met with condemnation from Messigen (who in turn wrote La Transfiguration in Latin as a protest). Nonetheless, the creation of colour which was integral to the makeup of his music was not useful for audiences either. Synaesthesia is not unique condition (it has been estimated to affect 4.4% of the UK population) and has affected plenty of musicians in history, namely Rimsky-Korsakov and Liszt, but Messiaen was under the impression everyone would see colour in a similar way to him. He wasn't wrong either, because there is scientific evidence to suggest a much higher percentage of the population have at least partial synaesthesia, but it wears off as the body develops. However, how could Messigen achieve the éblouissement he wished to achieve in listeners without the audience seeing colours whilst listening to his music? The key lay in his programme notes, which he started to increasingly write for his own works to truly educate the audience and enable his music to take effect. The titles of his pieces did as much to open the eyes of the audience to Catholicism as much as the music they heard. We are fortunate today that Messiaen wrote down and explained so

many of his thought processes and compositional techniques and methods.

Olivier Messiaen was not a man who wanted to write religious works, but a devoutly religious man who wanted to express everything he could about God. The notion of spirituality in his music was by no means an accident or mystical, it was at the forefront of his mind when composing. In my opinion, this is the first influence that sets him apart from his contemporaries in the 20th century. Messiaen was certainly not the first or last religious composer in the social history of music, and neither was he the only composer of the time writing about (or with) faith. Benjamin Britten displays spiritual elements in his War Requiem and Arnold Schoenberg also portrays evident Jewish ideas in his fragment opera Moses und Aron. On the other hand, whilst Messiaen composed for religion's sake, other composers used religion to compose more effectively for a different cause. Britten was agnostic, so the spiritual parts of his requiem were if anything a way of expressing anguish at the end of the Second World War in a medium that listeners could easily empathise with and tap into. He was not trying to depict religion but instead sentiments that he thought could be reached with religious musical ideas. In addition, the use of a Requiem as the form was more a form of bad-tempered irony considering it was full of juxtapositions between opposites (such as the tritone and use of poetry). Britten was a pacifist, therefore no matter how hard one looks for the religious links (of which most are ironic), this work was still a political one at heart. The Mass is a specific symbol of Britten's pacifist attack on the act of war and our necessity for religion. In a similar fashion, the writing of Schoenberg with Jewish influences is more likely also to have a political message behind it. In his case it was often anger at the Nazi State for their treatment of Jews. Therefore, Messiaen was a unique force in this turbulent period because his religious output treated spirituality as the primary source and Catholicism was his first agenda when writing; whereas other compositions of the time with religious elements often had a hidden agenda which they masked with religious intent.

Additionally, the principles of his religion Messiaen were attempting to convey were unique not only in the 20th century but throughout music history. As I previously mentioned, spiritual composers such as Bach wrote with a biblical text in mind and attempted to depict this text for a church congregation. This idea of expressing a specific truth of faith can be applied to Schoenberg as well, because he wrote for a liturgical purpose in the Jewish faith as well (Kol Nidre). However, Messiaen is distinct once more because although he does wish to depict a fact of Catholicism, this motivation is secondary for him behind the initial subconscious feelings of the listener. The Christian element of his works cannot be satisfactorily explained by description in terms of representation like his



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predecessors and contemporaries; instead it is achieved through thematization and symbolism. Messiaen created a new definition to attach to the title of "Religious Composer".

Stravinsky scathingly remarked of Messiaen "All you need to write like him is a large bottle of ink" and it is not hard to hear why he thought that. Underneath the surface of the music though, there were a plethora of influences on Messiaen of which Catholicism was the most significant. What others see as a mass of black ink instead has a complex methodology behind it, designed to portray God. The fact his audiences couldn't translate his difficult language didn't stop him from writing more eithershowing his religion to be more important than public recognition. Olivier Messiaen does fit the modernist mould, proven by his rejection of neo-classical composers who he said were "lazy" for following the work of Stravinsky. However, the question then remains as to what the distinction is between him and others who advocated musical progress. While John Cage and Arnold Schoenberg had method in their respective composition styles (Cage with music of chance and Schoenberg with twelve-tone serialism), both found those methods simply through defiance. Messiaen on the other hand built his method for composing progressive music primarily upon his religion.

Is Phaedra to blame for her downfall? A comparative analysis between Euripides *Hippolytus*, Seneca's *Phaedra* and Racine's *Phèdre*

Across all three plays, Phaedra is always seen to be set on fire by her fervent lust for Hippolytus, which leads her to act wildly and ferally. In Seneca's play, Phaedra wants to be like an Amazonian to achieve her goal:

Phaedra: "colla perfundant comae, umerosque summos, cursibus motae citis ventos sequantur, laeva se pharetrae dabit"

Thang Tu

Phaedra: May my hair flow over my neck, and over my high shoulders, may my hair, follow the winds as it moves as I swiftly run, my left hand will give itself over to the quiver (lines. 394-396)

Phaedra is suddenly no longer the daughter of Minos, but an Amazonian woman, yet it is all fake. It is clear to see that the sole reason for doing this is to pursue Hippolytus' love. She now resorts to publicly embarrassing herself, losing all the things that make her the queen of Troezen. The jussive subjunctives here make it seem as if it was an act of pleading, which goes against her reputation as the queen of Troezen and wife of Theseus, the slayer of the Minotaur. This aggression, not expected for someone of her social stature, makes her lose all the dignity she had, right in front of her Nurse and Chorus.

Not only in Seneca do we see her wild nature, but also in Racine. In Acte II Scène V, Phaedra famously confronts Hippolytus with her nurse Oenone by her side, when she says "Voilà mon cœur : c'est là que ta main doit frapper." (Here is my heart, there your hand must strike. line. 704). Surrendering to her stepson, Phaedra has been consumed by her "fureur" (fury/craze). She lays bare her breast

A SUMMARY OF THE PLOT

Phaedra and Racine's Phèdre.

There is the same core plot across all three tragedies. Venus has sent a sickness to Phaedra, second wife of Theseus and daughter of Minos, which makes her fall in love with Hippolytus, her stepson. Due to his commitment to chastity, Hippolytus rejects Phaedra's incestuous advances. Phaedra's nurse then takes control, when she puts a plan to tell confront Hippolytus. When Theseus arrives, under the false accusation of assault from Phaedra, Hippolytus is sent to exile and wished dead by one of his wishes granted by Neptune. Phaedra turns to commit suicide, Hippolytus dies to a sea monster and Theseus witnesses the breaking down of his household.

Tragedy has been at the heart of Western dramatic

arts, aiming to analyse the human mind, society and

aspects and reasons in the downfall of someone of

fears. Through the different representations of Phaedra in the three plays, we see the different

great reputation. However, it is important to

considered in this comparative essay. The

Cambridge English Dictionary² defines it as '(something that causes) the usually sudden

destruction of a person, organization, or

consider the definition of 'downfall' that is to be

government and their loss of power, money, or

reputation, dignity and the household. Thus, in this essay, I shall aim to discuss whether Phaedra is really

responsible for her downfall through the comparison

health'. To this definition, I also add the loss of

of three plays: Euripides' Hippolytus, Seneca's

Love and Lust

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¹ Throughout this essay, to avoid confusion and keep consistency, I shall be calling Racine's Phèdre Phaedra and Hippolyte Hippolytus. Also, I shall be using Roman terms for the mythology and gods.

² Carebridge English Digtig and bitter (/digtig and complexidge are /digtig and good).

² Cambridge English Dictionary: https://dictionary.cambridge.org/dictionary/english/downfall

waiting to be killed by Hippolytus. Phaedra cries out in pain and passion and she is inflamed with a burning love for Hippolytus. Two lines later she calls on Hippolytus to "Frappe!" (*Strike!*). The imperative, along with the break in the rhythm of the alexandrine, is almost cathartic for her. Everything she has said has built up to this moment, where she wants to be struck by the sword of Hippolytus. This shocks us. So far in Racine's play we have seen how even the mention of his name causes her pain. In Acte I, Scène II, when Oenone says "Cet Hippolyte" (*line. 205*), Phaedra responds with "Ah, Dieux!", however in his presence Phaedra's "fureur" is even stronger, to the extent that this mental weakness drives her wild.

Nevertheless, her wild nature is not only from her actions, but her desire for sexual activity. Again, in Racine's Acte II, Scène V, the imperative of "Frappe!", in context, has strong sexual connotations. Moreover, after Hippolytus does not obey her, she demands his sword and, in a quasimasturbatory manner, snatches it. She desires sexual interaction with her stepson. This is also seen in Euripides' Hippolytus, in line 222, with " ἐπίλογχον ἔχουσ' ἐν χειρὶ βέλος" ([I wish] to hold in my hand a barbed spear). Understandably "βέλος" (barbed spear) can be taken as a phallic metaphor, but in this context, now she desires to be the hunter. Hippolytus adores hunting, but Phaedra takes Hippolytus' pastime and makes it hers. Consumed by lust, she is a huntress, but rather than imitating Hippolytus' idol Diana, she follows in the footsteps of Cupid. She would rather hunt for love than for beasts, a cruel irony considering she is almost feral in her quest for sexual satisfaction.

Fate and Destiny

Despite this, the reason for Phaedra's downfall does not only centre on herself, but many others. Of these, her ancestry and heritage play a large part. Firstly, in Racine's Phèdre, Phaedra is called by Hippolytus "La fille de Minos et de Pasiphaé" (the daughter of Minos and Pasiphaë, line 36). Pasiphaë, the mother of Phaedra, famously gave birth to the Minotaur after she had intercourse with a bull. Ovid also mentions in his Heroides³, Phaedra's lineage is also said to extend back to Jupiter with Europa, where Jupiter disguised himself as a bull. Hence, not only does Phaedra's bloodline contain lovers who have been seduced unnaturally, but also by nature. Phaedra's paradoxical passion is not only unnatural as it is incestuous, but natural as well since she is so inflamed with love for Hippolytus, whose greatest love is to hunt, shown when he opens the play hunting with his companions. Moreover, her Cretan heredity has made her fall in love with Hippolytus. Phaedra recognises this saying: "quid furens saltus amas? fatale miserae matris agnosco malum." (What is this passion for the forest? I recognise the fatal evil of my wretched mother, lines 113-114)

Theseus and Abandonment

However, despite his heroics, Theseus' role in her downfall must be analysed, for he is also key to this question. Although Seneca's version seems to portray Phaedra as the lustful stepmother and the sole responsibility for her downfall, the context must be considered as well. For, whilst the scenes take place at Troezen, Theseus has been missing for four years already. In Racine's play, Theseus is even rumoured to have died and in Euripides' Theseus returns in the middle of the tragedy from visiting the Oracle at Delphi. The common theme throughout these three plays is the lack of Theseus for Phaedra, which is partly the reason she has such a fall from grace. Since he has gone, Phaedra is left with essentially a love vacuum, which can only be filled by Hippolytus. In Racine's story, Phaedra says that "II n'est point mort, puisqu'il respire en vous." ([Theseus] is not dead, since he breathes within you, line. 627). Despite our impressions, Phaedra is still good natured within, as she is still in love with Theseus and she will not let go of that. Phaedra is also seen to be scornful of him, as she says, "profugus en coniunx abest praestatque nuptae quam solet Theseus fidem" (Behold, my fugitive husband is gone, and how Theseus upholds his marriage vows which he was wont to hold, line. 91). In a very sarcastic tone, her passion for Hippolytus does not only stem from her own ancestors, but also that she has been wounded emotionally by abandonment from Theseus for years.

Divine Will and the gods

Yet there is a reason larger than her ancestry itself: the gods, and in particular Venus. No play is this more prevalent than in Euripides', which starts off with a prologue, where Venus tells the audience her plan. She plans to punish Hippolytus for admiring

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³ Heroides IV, *Phaedra Hippolyto*, lines 55-56

Diana more than herself by makina Phaedra fall in love with him, so that he may die by Theseus' wishes. We question whether Phaedra's passion for Hippolytus was ever her free choice. Both here and in Racine, where Phaedra says in line 249 "O haine de Vénus!" (Oh, Venus' hatred!). Venus has made Phaedra fall in love, and thus Venus is culpable for Phaedra's downfall. Moreover, Seneca's Phaedra sees herself in lines 181-183 "sic, cum gravatam navita adversa ratem propellit unda, cedit in vanum labor et victa prono puppis aufertur vado" (as when a sailor drives forth a burdened ship onto unfavourable waves, his work falls to nothing and his ship, conquered by the rushing tide, is taken away). Phaedra knows that even if she does everything possible to change it, her fate will always be to love and her "vicit ac regnat furor" (my raving conquers and reigns, line 184). Her battle with her "furor" (craze) is her perennial quest, however, much like Sisyphus, whatever she does, she will not succeed for she is subject to the gods and their desires, not her own. I would also like to draw attention to the word "fureur" again, for it only emphasises her helplessness even more. Larousse takes it to mean an "impetuous violence from natural elements4". Although she may be raving wildly, this is not due to her own character but from natural sources. Her behaviour is not of her own accord, but from the gods', and more importantly Venus', wishes.

Self-Control and Sophrosyne

Within Phaedra's speeches in all three plays before her death, an important issue comes up: Self-control versus Fate. In Euripides' Hippolytus, Phaedra's final words are "τῆς νόσου δὲ τῆσδἑ μοι κοινῆ μετασχών σωφρονεῖν μαθήσεται." (By sharing in this affliction of mine, he will learn what self-control is, lines. 730-731). There is a sinister, almost vengeful undertone in this sentence of hers, seen in "μαθήσεται" (he will learn) showing some of her wicked traits, but what is more prevalent is the sarcastic nature of her use of the word "σωφρονεῖν" (self-control). Although here it is translated as self-control, sophrosyne's meaning can range from chastity to prudence. For Phaedra, sophrosyne is to be able to take control of all your emotions, not just your sexual desires. The selfcontrol here is the importance of facing and controlling your fears. Phaedra already knows that she is to die, but Hippolytus, thinking that Diana will always save him, does not. Phaedra is not to teach

Hippolytus about chastity, but about the reality of fate. Sophrosyne is not to stay chaste, but to realise your fate, to accept it and to not fight against it.

Despite first impressions, Seneca also portrays Phaedra in a positive light. Phaedra's final words must be given close attention. She cries out: "mucrone pectus impium iusto patet cruorque sancto solvit inferias viro." (my unfaithful heart must now suffer the sword of righteousness, and my blood releases the sacrifices for a man who could never sin, lines. 1197-1198). She now, much like in Euripides' play, realises her fate. She must give in to what is right, and in this context her punishment for her crime of incest is death. The juxtaposition of "impium" (unfaithful) and

"iusto" (*righteous*) represents Phaedra's character. She is torn apart by her lust for Hippolytus, yet inside she is still in control of her emotions such that she has not lost all her reputation and dignity.

In Phèdre, Racine does something that neither of the other two plays do. He decides to make Phaedra the one to tell Theseus the truth. She meets Theseus, saying, "Il faut rompre un injuste silence; il faut à votre fils rendre son innocence." (I must break this unjust silence; I must restore your son's innocence, lines. 1617-1618). The anaphora of the "il faut" (I must) emphasises Phaedra's wish to be morally right. She knows that throughout this play she has been acting scandalously, but now, with Hippolytus dead, she has an epiphany. To uphold her reputation, she must do Hippolytus justice. Later she goes on to say, "Le fer aurait déjà tranché ma destinée; mais je laissais gémir la vertu soupçonnée." (The blade would have already cut my fate, but I let suspected virtue cry out for you. lines. 1633-1634). What is interesting here is that, much like in Euripides' play, the question of free will comes up. Racine mentions "le fer" (the blade), an allusion to the three fates spinning the threads of mortals' lives. Phaedra recognises her fate is at an end, but she would rather let "gémir la vertu" (virtue cry out). This mental strength, to go against the fates, must be noted. With Phaedra's final appearance, we are left with her bravery and courage and virtue as she dies to poison.

In this essay, I have tried to outline how both Phaedra and other people are to blame for her downfall. What is key across all three plays

⁴ Littéraire. Violence impétueuse des éléments naturels – Larousse

however is the importance of self. In Euripides' play, the focus is largely on the gods, since it has been clear from the very start that Venus is to blame for Phaedra's death. For Seneca, it is her own sexual desires and her behaviour that go against Seneca's stoic philosophy that drive her to suicide and in Racine's version Oenone, the Nurse, along with the gods are the major factors in Phaedra's downfall. Nevertheless, although it can be argued that Phaedra's downfall is the result of her beastly, libidinous and cowardly traits, the more important issue is that of Self-control versus Fate. Phaedra, despite her sophrosyne and mental capacity, is still the victim of a power that is greater than she is. It has been destined for her to end her life in this way, bringing up a crucial question: did Phaedra ever have the free will to live her life? Throughout all three plays, she has been surviving this illness with the constant knowledge that Venus has caused her all this pain. Phaedra has never been in total control of her emotions, but she has always been able to face her fears and understand that she cannot, and never will be able, to change her fate. This quasiheroic act, to go fearlessly into death and to accept fate, is what saves her from her judgement in the Underworld by her father Minos. To explicitly say that Phaedra has a complete downfall is to not state the truth, for in the end her emotional maturity comes through and so does her reputation and dignity.



From the Headmaster

I am often asked what we, as a school, understand by scholarship, what being a true scholar really means. In the twenty-first century both of these terms are clearly about so much more than simply the acquisition of knowledge. Knowledge is readily available with the mere touch of a screen, often on devices less than half the size of a traditional paperback book. In which case, scholarship must be more about an attitude, a mind-set, a set of skills.

As one of our six School Values we define scholarship as the growth of intellectual curiosity, independence, creativity, innovation and habits of learning within a scholarly community through inspirational subject specialists who provoke and stretch the students' minds and inculcate a lifelong love of learning. The Journal 2019 goes some way to proving that far from an ideal this is, in fact, reality. True scholarship demands students to travel outside their comfort zone, to challenge conventional thought, and to embrace an openness to unfamiliar experiences and topics. Rather than seen as a succession of obstacles, true scholarship should relish perseverance and should constantly pursue new ways to expand horizons and capabilities. Most importantly, however, and as I read through these extraordinary essays this comes through particularly strongly, scholarship is about passion and enjoyment. A quotation by Carl Friedrich Gauss sums this up concisely: "It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment".

I would like to congratulate Mr Bradford, our Head of Scholarship, as well as the committee which has worked tirelessly putting this publication together and, of course, all those students whose inspiring works are included. I hope you enjoy reading these essays.

Dr Jon Cox Headmaster Royal Grammar School Guildford



'Quiet people have the loudest minds.' Stephen Hawking



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